8. MASS AND INERTIA

Performance, stability, control, and strength analyses of airborne vehicles depend not only on the mass of the vehicle but on the distribution of the mass within the vehicle. This concept of mass distribution is reflected in the property of the vehicle called moment of inertia. This Section discusses moment of inertia determination for two types of airborne vehicles — manned aircraft and missiles.

8.1 AIRCRAFT MASS AND INERTIA

The purpose of this Section is to furnish the engineer with a method for rapidly but accurately estimating the moment of inertia of manned aircraft during the preliminary-design period. Such inertias are needed in order that dynamic load and stability characteristics of the aircraft may be evaluated.

The following pages present basic moment-of-inertia theory, a discussion of inertia methods in general with the assumptions and conclusions used in evolving the Datcom method, and a discussion of the Datcom method in detail, with a summary showing the step-by-step procedure to follow when using this method.

This method applies to all existing combat and transport aircraft including those of unconventional* design. If radically different airplane configurations evolve, the present methods will have to be altered. The tools needed are a weight-and-balance statement, a three-view drawing, and some knowledge of the design characteristics of the airplane. A total time of approximately three hours for one man is needed to estimate inertia by this method, and the accuracy obtained is within the tolerance required for any preliminary-design project.

Basic Moment-of-Inertia Theory

Moment of inertia is the measure of resistance to angular acceleration, as mass is the measure of resistance to linear acceleration. Moment of inertia may be mathematically derived as follows:

If torque is expressed as the product of force and radius

$$T = Fr$$

and the following substitutions are made:

$$F = ma \quad \text{and} \quad a = \alpha r$$

then

$$T = mar$$

or

$$T = mr^2\alpha \qquad 8.1\text{-a}$$

where

- a is the linear acceleration
- α is the angular acceleration
- m is the mass

The term mr^2 is defined as the moment of inertia (I) and equation 8.1-a may be written

$$T = I\alpha \qquad 8.1\text{-b}$$

If a body of mass m is caused to rotate about a remote axis y (see sketch (a)) the following relationship exists:

$$I_y = mr^2 = m\,(x^2 + z^2) \qquad 8.1\text{-c}$$

However, since mass m not only offers resistance to rotation about the y axis but also offers resistance to rotation about its own centroidal axis, the total inertia of m about y is

$$I_y = mr^2 + I_{oy} \qquad 8.1\text{-d}$$

where I_{oy} is the inertia of m about its own centroidal axis.

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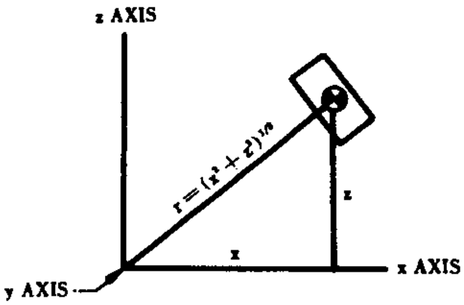
*The term unconventional as used herein refers chiefly to extreme locations of the wing, fuselage, tail and power plant sections with respect to each other, and to the size and mass of these sections. However, when the shape and mass distribution of any of the sections change considerably over present state of the art design, the method for computing inertia as described herein should be altered accordingly.



SKETCH (a)

If mass m is divided into several parts, $m_1, m_2, \ldots m_n$, then the total inertia of the sum of these parts about y is

$$I_y = m_1 r_1^2 + I_{oy_1} + m_2 r_2^2 + I_{oy_2} + \ldots m_n r_n^2 + I_{oy_n}$$

or

$$I_y = \sum_{i=1}^{n} \left(m_i r_i^2 + I_{oy_i} \right) \qquad 8.1\text{-e}$$

Finally, if the total inertia about a remote axis is computed from equation 8.1-e, the inertia of the total mass about its own centroidal axis can be computed from equation 8.1-d, or

$$I_{oy} = I_y - m\bar{r}^2 \qquad 8.1\text{-f}$$

where $\bar{r}$ is the distance from the y axis to the centroid of the total mass.

$$\bar{r} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i} \qquad 8.1\text{-g}$$

Inertia Methods in General

As equation 8.1-d indicates, the inertia of a body about a remote axis depends on three basic factors: (1) mass, (2) the distance of the mass from the remote axis, and (3) the inertia of the mass about its own centroidal axis. If any one of these factors is ignored, the inertia derived will not be accurate.

A common method is to divide the airplane into many sections, so that the I_o values may be calculated easily and accurately. The total airplane inertia about the remote axis may then be derived by equation 8.1-e, and the total inertia about the aircraft's centroidal axis by equation 8.1-f. Unfortunately, however, this method requires a detail breakdown of the masses and centroids of the components of the aircraft as well as a large time expenditure. Therefore the method does not lend itself to successful application at the preliminary design level.

Another method for computing inertia is one in which the total mass and dimensional data of the aircraft are used as parameters in an empirical inertia equation. However, the equations have to be based on aircraft with mass distribution similar to that of the proposed aircraft, since the mass and locations of the wing, fuselage, tail, and engines can vary greatly between aircraft. This means that a large amount of statistical data has to be available, and this type of method is almost useless for aircraft of unconventional design, where there is a lack of statistical data.

Obviously neither of these two methods fulfills the requirements for one rapid but accurate method independent of airplane type or conventional nature. But consideration of a few facts about inertia methods in general shows which method is preferable in a particular situation. These facts are:

(1) When statistical data are not available —

(a) The time required to compute inertia and the accuracy of the answer are directly proportional to the number of sections into which the airplane is divided.

(b) The number of sections into which the airplane is divided is dependent on the amount of detail information available.

(2) When statistical data are available —

(a) Time is inversely proportional to parametric correlation, which is the mutual relationship of the inertia of the proposed aircraft to statistical data by means of parameters such as size, shape, and mass.

(b) Accuracy is dependent upon the extent to which parametric correlation is applied. This means that, if these correlations are carried beyond the bounds of mass distribution similarity, the accuracy is affected adversely; if carried only as far as these bounds, the accuracy is affected only slightly.

Therefore the method evolved is one that divides the airplane into the least number of sections necessary to maintain sufficient mass distribution similarity to permit valid parametric correlation. Thus the accuracy obtained is within the tolerance required for preliminary-design studies. After careful consideration, these five major sections were chosen:

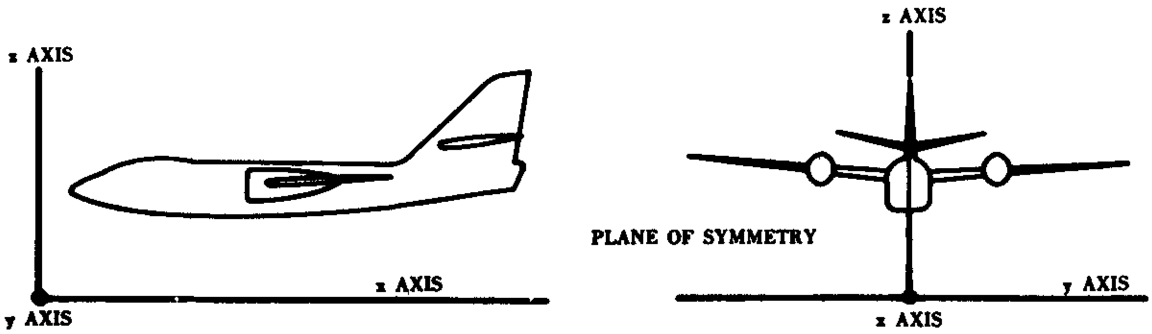
(1) Wing
(2) Fuselage
(3) Horizontal Stabilizer
(4) Vertical Stabilizer
(5) Power Plant (Engine and Nacelle)

The inertias for fuel, cargo, and other variable items may also be added, as shown in the sample problem.

DATCOM METHOD

The method consists of determining the mass and centroids for each of the major sections and then, by equations involving the parameters of size, shape, mass, and centroids, calculating the I_o values for each of these sections. Once this is completed, the inertias about the remote axis may be derived by equation 8.1-e, and hence the inertias about the airplane centroids by equation 8.1-f.

The I_o formulas are based on aircraft in a gear-up configuration with expendable and variable items, such as fuel, cargo, and passengers, deleted. Since it is impossible to predict the aircraft configurations for which inertia will be needed, the inertia for the expendable and variable items comprising these configurations must be added to the basic inertia derived from the method.



SKETCH (b)

Step 1. Selection of remote axes — Three remote axes are chosen so that inertia in pitch, roll, and yaw may be calculated. The origin of these axes should be located so that distances to the centroids of the major sections are positive and should be located on the plane of symmetry of the airplane. This type of location simplifies calculations and minimizes errors. An example of a remote-axis selection is shown in sketch (b). The "x" axis is the axis along which longitudinal distances are measured; the y axis is the axis along which the lateral distances are measured; the z axis is the axis along which vertical distances are measured.

Step 2. Mass and centroid determinations for the major sections — With the aid of a weight-and-balance statement, the three-view drawing, and a knowledge of the locations of the group items, the mass and corresponding centroids for each of the five sections are determined. These values are then recorded on a form similar to the one shown in the sample problem.

For single-engine aircraft with the engine mounted on the aircraft plane of symmetry and with the nacelle structure in part integral with the fuselage, only the engine is considered as a separate section, and the nacelle and other engine items are considered with the fuselage section. Only x and z distances are recorded for all sections mounted on the aircraft plane of symmetry. However, for noncenterline-mounted vertical stabilizers and power-plant sections y distances are recorded in addition to x and z values, since the I_o values are calculated about the centroids of each section. For example, the power-plant sections for a four-engine aircraft are analyzed as follows:

The masses of both inboard power-plant sections are added together and recorded. The lateral distance from the aircraft plane of symmetry to the centerline of one of the inboard power plants is recorded along with the distances from the other two axes. The outboard power plants are analyzed similarly, except that the lateral distance recorded is measured from the aircraft plane of symmetry to the centerline of one of the outboard power plant sections.

Care must be taken to use consistent units for mass and distance throughout the entire calculation. Since most group components are listed by pounds, this unit of weight may be used for all calculations. The resulting inertias may then be converted to units involving mass by dividing by the acceleration due to the force of gravity at the desired altitude. An example of such a conversion is included in the sample problem.

Step 3. Calculations of I_o for the major sections — The I_o values for the major sections are determined by first considering an "ideal" formula that closely correlates with the shape of the section. These formulas are labeled "ideal," since they are based on sound mathematical principles, a prerequisite for any school, and since they assume a homogeneous mass distribution throughout the section. The result from the "ideal" formula is then multiplied by a K factor that accounts for deviations in the homogeneous nature of the mass of the section. These factors are based on statistical data. It is found that for some sections a constant factor may be used. For other sections, where the mass distribution may vary considerably, it is found that the K factor varies primarily with the centroid location of the section. Graphs showing the variable K factors, along with substantiating correlation plots, are included as figures. The I_o calculations for each section follow:

(a) Wing Pitching I_o

The wing pitching I_o formulas are listed in the summary. The formulas are based on a consideration of three basic wing shapes (see figure 8.1-22). A constant K factor ($K_o = 0.703$) is used for all wing designs.

(b) Wing Rolling I_o

Because of the large span of the wing with respect to other sections of the aircraft, the wing rolling I_o calculations has the greatest effect on the rolling inertia of the aircraft. The variable K factor (K_1) shown in figure 8.1-23 is based primarily on the lateral centroid of half the wing. As this centroid approaches the aircraft plane of symmetry, more weight must be concentrated in the inboard section of the wing, thereby lowering its I_o value.

(c) Wing Yawing I_o

As is shown by statistical data and by an analysis of the inertias of flat plates, the wing yawing I_o is equal to the sum of the wing pitching and rolling I_o's.

(d) Fuselage Pitching I_o

The fuselage pitching I_o formula is based on that of a combination cylindrical shell and conical shell. The formula provides that as the ratio of the fuselage wetted area to the theoretical wetted area of the fuselage as a two-way cone increases, the inertia approaches that of a cylindrical shell. The fuselage I_o is probably the most difficult to correlate parametrically, since parameters are not available to accurately predict the location of large mass items such as landing gear, electronic equipment, etc. However, statistical data show that the longitudinal centroid location has a definite bearing on the inertia, and therefore the variable K factor (K_3) uses this centroid location as its basic parameter (see figure 8.1-24). It should also be noted that the fuselage pitching I_o has the greatest effect on the pitching inertia of the airplane.

(e) Fuselage Rolling I_o

The formula is based on that of a cylindrical shell of an average diameter determined by consideration of the fuselage wetted area. The fuselage diameter and the ratio of the fuselage structural mass to the total fuselage section mass are the parameters by which K is computed. As the fuselage diameter decreases, the I_o approaches that of a solid cylinder, since the solidity of the equipment items is more effective. Therefore the fuselage diameter is directly proportional to the I_o value since the rolling I_o of the solid circular cylinder is less than that of a cylindrical shell.

The ratio of the fuselage structural mass to the total fuselage-section mass is also directly proportional to the I_o result, since as the value of the ratio decreases, the effect of the solidity of equipment items increases, which decreases the final I_o result. The graph for determining the variable K factor (K_4) is shown as figure 8.1-25.

(f) Fuselage Yawing I_o

As is shown by statistical data and by an analysis of the inertias of cylindrical bodies, the fuselage yawing I_o is equal to the fuselage pitching I_o.

(g) Tail Section I_o

The tail section I_o determinations are similar to those discussed for the wing. Note that the constant K factor used in evaluating tail section I_o's differs from the wing K_o value. For horizontal or vertical stabilizers $K_o = 0.771$.

(h) Power-Plant Pitching and Rolling I_o

The power-plant pitching and rolling formulas are based on that of a solid circular cylinder. The pitching I_o formula accounts for differences in length between the nacelle structure and the engine. Both formulas include a constant factor that may be used for all designs. These formulas are given in the Datcom Summary.

(i) Power-Plant Yawing I_o

As is shown by statistical data and an analysis of the inertias of cylindrical bodies, the power plant yawing I_o is equal to the power plant pitching I_o.

Step 4. **Total airplane inertia — When the data discussed above and similar data for the expendable or variable load items have been itemized, the total airplane inertias in pitch, roll, and yaw can be determined from equations 8.1-e and 8.1-f. For further illustration, see the sample problem.**

Ten aircraft were analyzed in detail, in order to substantiate the method and the choice of major sections. These aircraft were chosen for their availability of data, for the degree in which they could be analyzed in detail, and for the large cross section of configurations, which include (1) those with a weight empty of from 7,000 to 125,000 pounds, (2) swept and nonswept wing designs, (3) combat, cargo, and passenger types for Air Force, Navy, and commercial uses, (4) reciprocal and jet, both multiple- and single-engine designs, (5) wing and fuselage engine locations, (6) wing and fuselage main-landing-gear locations, (7) fuselage and nonfuselage fuel locations, and (8) all types of tail configurations, including those having wing-mounted elevons instead of horizontal stabilizers.

DATCOM METHOD — SUMMARY

1. Notation*

I_y	pitching moment of inertia about a remote axis
I_x	rolling moment of inertia about a remote axis
I_z	yawing moment of inertia about a remote axis
I_{oy}	pitching moment of inertia about the centroidal axis of the body
I_{ox}	rolling moment of inertia about the centroidal axis of the body
I_{oz}	yawing moment of inertia about the centroidal axis of the body
W_w	weight of wing section including wing carry-through structure
$\bar{y}_w$	lateral centroidal distance of half-wing from aircraft plane of symmetry
C_a, C_b, C_c	wing parameters measured parallel to plane of symmetry (see figure 8.1-22 and page 8.1-7)
c_r	root chord of wing (at ℄)
c_t	tip chord of wing
Λ_{LE}	sweepback angle of wing leading edge
W_f	weight of fuselage section
W_{fs}	weight of fuselage structure
$\bar{x}_f$	longitudinal centroidal distance of fuselage from nose
l_B	length of fuselage
d	average maximum diameter of fuselage $= \dfrac{\text{max. diameter} + \text{max. width}}{2}$
S_s	fuselage wetted area
W_H	weight of horizontal stabilizer section
$\bar{y}_H$	lateral centroidal distance of half horizontal stabilizer from aircraft plane of symmetry
c_{r_H}	root chord of horizontal stabilizer (at ℄)
c_{t_H}	tip chord of horizontal stabilizer
b_H	span of horizontal stabilizer
Λ_{LE_H}	sweepback angle of horizontal-stabilizer leading edge
W_V	weight of vertical stabilizer
$\bar{z}_V$	vertical centroidal distance of vertical stabilizer from theoretical root chord (at fuselage)
c_{r_V}	root chord of vertical stabilizer (at fuselage)

* Since the values used for W in this section are those of weight instead of mass, the solution of the equations is more general and applicable to any altitude. Consequently, the inertias throughout the problem are in lb-in.^{2}, but are converted at the end of the problem to slug-ft^2 by using the value of gravity at the particular altitude.

c_{t_V} tip chord of vertical stabilizer

b_{v_1} span of vertical stabilizer (tip to fuselage)

Λ_{LE_V} sweepback angle of vertical stabilizer leading edge

W_P weight of power plant section

W_e weight of engine and propeller (if applicable)

l_e length of engine including propeller (if applicable)

d_e average maximum diameter of engine

l_p length of nacelle structure

ρ ratio of weight to chord for wing shapes

2. Select three remote axes and an origin location for these axis.

3. Determine mass and centroids for all sections and applicable expendable and variable load items and record on a convenient form.

4. From the information determined by item 3, calculate and record mass times centroid and mass times centroid squared.

5. Determine the I_o figures for the major sections as follows:

a. Wing pitching I_o (see figure 8.1-22 for development of equations)

if: $$W_W = \frac{\rho}{2}(-C_a + C_b + C_c) \quad 8.1\text{-h}$$

$$W_W x = \frac{\rho}{6}(-C_a^2 + C_b^2 + C_c C_b + C_c^2) \quad 8.1\text{-i}$$

$$I = \frac{\rho}{12}(-C_a^3 + C_b^3 + C_c^2 C_b + C_c C_b^2 + C_c^3) \quad 8.1\text{-j}$$

$$I_{oy} = K_o\left[I - \frac{(W_W x)^2}{W_W}\right] \quad 8.1\text{-k}$$

where:

$K_o = 0.703$

C_a is the smallest of the following values:

$$c_r \quad ; \quad \frac{b_W \tan \Lambda_{LE}}{2} \quad ; \quad c_t + \frac{b_W \tan \Lambda_{LE}}{2} \quad 8.1\text{-}l$$

C_b is the intermediate value

C_c is the largest value

b. Wing Rolling I_o

$$I_{ox} = \frac{W_w b_W^2 K_1}{24}\left(\frac{c_r + 3c_t}{c_r + c_t}\right) \quad 8.1\text{-m}$$

where K_1 is obtained from figure 8.1-23.

c. Wing Yawing I_o

$$I_{oz} = I_{oy} + I_{ox} \quad 8.1\text{-n}$$

d. Fuselage Pitching I_o

$$I_{oy} = \frac{W_f S_s K_2}{37.68}\left(\frac{3d}{2\ell_B} + \frac{\ell_B}{d}\right) \qquad 8.1\text{-o}$$

where K_2 is obtained from figure 8.1-24.

e. Fuselage Rolling I_o

$$I_{ox} = \frac{W_f K_3}{4}\left(\frac{S_s}{\pi \ell_B}\right)^2 \qquad 8.1\text{-p}$$

where K_3 is obtained from figure 8.1-25.

f. Fuselage Yawing I_o

$$I_{oz} = I_{oy} \qquad 8.1\text{-q}$$

g. Horizontal Stabilizer Pitching I_o

Use same equations as wing pitching I_o

$K_o = 0.771$

h. Horizontal Stabilizer Rolling I_o

$$I_{ox} = \frac{W_H b_H^2 K_4}{24}\left(\frac{c_{r_H} + 3c_{t_H}}{c_{r_H} + c_{t_H}}\right) \qquad 8.1\text{-r}$$

where K_4 is obtained from figure 8.1-26.

i. Horizontal Stabilizer Yawing I_o

$$I_{oz} = I_{oy} + I_{ox} \qquad 8.1\text{-s}$$

j. Vertical Stabilizer Rolling I_o

$$I_{ox} = \frac{W_v b_{v_1}^2 K_5}{18}\left[1 + \frac{2 c_{r_V} c_{t_V}}{(c_{r_V} + c_{t_V})^2}\right] \qquad 8.1\text{-t}$$

where K_5 is obtained from figure 8.1-27.

k. Vertical Stabilizer Yawing I_o

Use same equations as wing pitching I_o (use twice the vertical stabilizer span as the value of b in the equations for wing pitching I_0)

$K_o = 0.771$

Vertical Stabilizer Pitching I_o

$$I_{oy} = I_{ox} + I_{oz} \qquad 8.1\text{-u}$$

m. Power Plant Pitching I_o

$$I_{oy} = 0.061\left[\frac{3}{4} W_p d_e^2 + W_e \ell_e^2 + (W_p - W_e)\, \ell_p^2\right] \qquad 8.1\text{-v}$$

n. Power Plant Rolling I_o

$$I_{ox} = 0.083\, W_p d_e^2 \qquad 8.1\text{-w}$$

o. Power Plant Yawing I_o

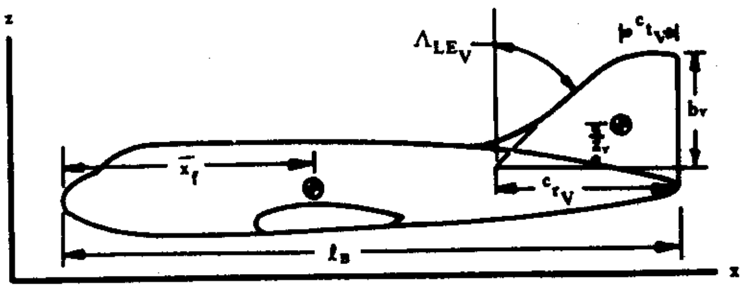
$$I_{oz} = I_{oy} \qquad (8.1\text{-}x)$$

6. Determine the I_o value for the expendable and variable load items by considering the conventional inertia formulas which closely match the shape of these load items. (See sample problem.)

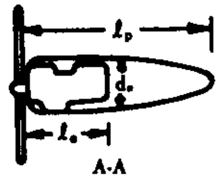
7. Determine the total airplane inertias in pitch, roll, and yaw by using equations 8.1-e and 8.1-f. (See sample problem.)

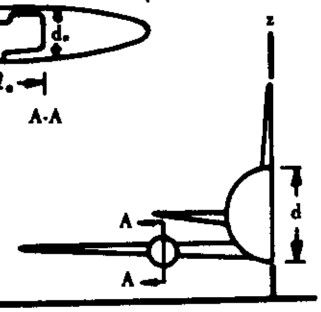
Sample Problem

Given:

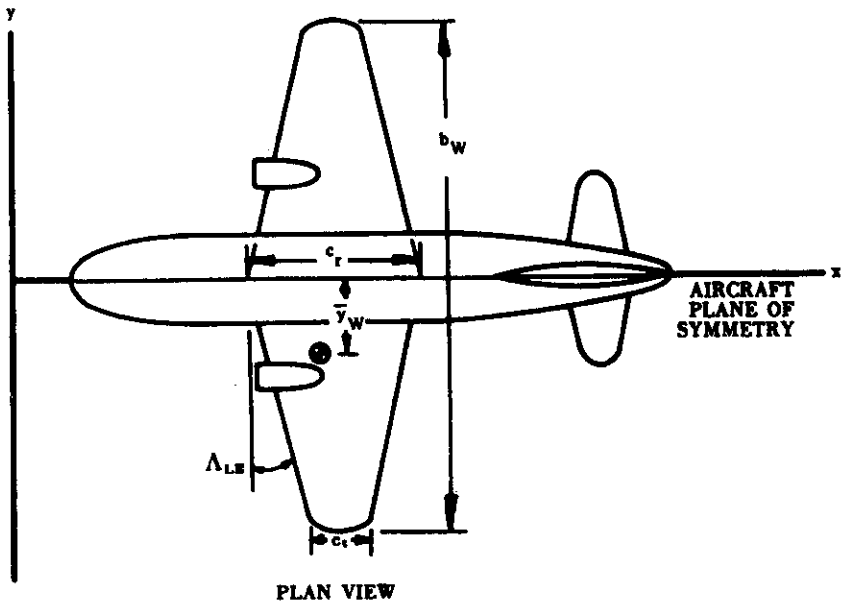


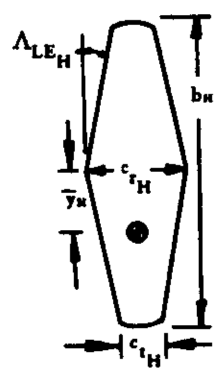
PROFILE VIEW





FRONT VIEW





PLAN VIEW

Wing

$\Lambda_{LE} = 12.1°$

$b_W = 10$

$c_r = 300$ in.

$c_t = 100$ in.

$\bar{y}_W = 150$ in.

Hor. Stab.

$\Lambda_{LE_H} = 12°$

$b_H = 400$ in.

$c_{r_H} = 100$ in.

$c_{t_H} = 50$ in.

$\bar{y}_H = 80$ in.

Ver. Stab.

$\Lambda_{LE_V} = 37°$

$b_{V_1} = 200$ in.

$c_{r_V} = 250$ in.

$c_{t_V} = 100$ in.

$\bar{z}_V = 75$ in.

Fuselage

$l_B = 1200$ in.

$d = 150$ in.

$\bar{x}_f = 500$ in.

$S_s = 400{,}000$ sq in.

$W_{fs} = 8000$ lb

Power Plant

$l_p = 200$ in.

$l_e = 100$ in.

$d_e = 50$ in.

$W_e = 7000$ lb

SECTION	WEIGHT (lbs)	x (in.)	z (in.)	y (in.)
Wing	15,000	650	150	
Fuselage	20,000	600	200	
H. Stab.	1000	1150	200	
V. Stab.	300	1200	300	
P. Plant	10,000	520	150	200
Fuel	20,000	650	150	
Cargo	10,000	500	200	

Compute:

1. Calculate and tabulate the products of weight and centroid location and the products of weight and centroid location squared.

2. Determine the I_o values for the major sections.

   a. Wing pitching I_o

$$C_a = \frac{b_W \tan \Lambda_{LE}}{2} = \frac{(1000)\tan 12.1°}{2} = 107 \text{ in.}$$

$$C_b = c_t + \frac{b_W \tan \Lambda_{LE}}{2} = 100 + 107 = 207 \text{ in.}$$

$$C_c = c_r = 300 \text{ in.}$$

(equation 8.1-l)

$K_o = 0.703$ (constant for any wing)

$$\rho = \frac{W_W}{.5\,(-C_a + C_b + C_c)} \quad \text{(equation 8.1-h)}$$

$$= \frac{15{,}000}{.5\,(-107 + 207 + 300)} = 75 \text{ lb/in.}$$

$$W_W x = \frac{\rho}{6}[-C_a^2 + C_b^2 + C_c C_b + C_c^2] \quad \text{(equation 8.1-i)}$$

$$= 12.5\,[-(107)^2 + (207)^2 + (300)(207) + (300)^2] = 2{,}293{,}750 \text{ lb-in.}$$

$$I = \frac{\rho}{l}[-C^{3} + C_b^3 + C_c^2 C_b + C_c C_b^2 + C_c^3] \quad \text{(equation 8.1-j)}$$

$$= 6.25 [\ (107)^3 + (207)^3 + (300)^2 (207) + (300)(207)^2 + (300)^3] = 413{,}308{,}750 \text{ lb-in.}^2$$

$$I_{oy} = K_o \left[I - \frac{(W_w x)^2}{W_w} \right] \text{(equation 8.1-k)}$$

$$= (0.703) \left[413{,}308{,}750 - \frac{(2{,}293{,}750)^2}{15{,}000} \right] = 43{,}976{,}971 \text{ lb-in.}^2$$

b. **Wing Rolling I_o**

$$\frac{\bar{y}_w}{\frac{b_w}{6} \frac{c_r + 2c_t}{c_r + c_t}} = \frac{150}{208} = 0.72$$

$K_1 = 0.67$ (figure 8.1-23)

$$I_{ox} = \frac{W_w b_w^2 K_1}{24} \left(\frac{c_r + 3c_t}{c_r + c_t} \right) \quad \text{(equation 8.1-m)}$$

$$= \frac{(15{,}000)(1000)^2(0.67)}{24} \left(\frac{300 + 300}{300 + 100} \right) = 628{,}125{,}000 \text{ lb-in.}^2$$

c. **Wing Yawing I_o**

$$I_{oz} = I_{oy} + I_{ox} \quad \text{(equation 8.1-n)}$$

$$= 43{,}976{,}971 + 628{,}125{,}000 = 672{,}101{,}971 \text{ lb-in.}^2$$

d. **Fuselage Pitching I_o**

$$\frac{\left| \frac{\ell_B}{2} - \bar{x}_f \right|}{\frac{\ell_B}{2}} = \frac{|\,600 - 500\,|}{600} = 0.17$$

$K_2 = 0.83$ (figure 8.1-24)

$$I_{oy} = \frac{W_f S_s K_2}{37.68} \left(\frac{3d}{2 l_B} + \frac{l_B}{d} \right) \quad \text{(equation 8.1-o)}$$

$$= \frac{(20{,}000)(400{,}000)(0.83)}{37.68} \left(\frac{450}{2400} + \frac{1200}{150} \right) = 1{,}442{,}807{,}855 \text{ lb-in.}^2$$

e. **Fuselage Rolling I_o**

$$\frac{(d)^{1/2}(W_{fs})}{W_f} = \frac{(150)^{1/2}(8{,}000)}{20{,}000} = 4.9$$

$K_3 = 0.97$ (figure 8.1-25)

$$I_{ox} = \frac{W_f K_3}{4} \left(\frac{S_s}{\pi l_B} \right)^2 \quad \text{(equation 8.1-p)}$$

$$= \frac{(20{,}000)(0.97)}{4} \left(\frac{400{,}000}{1200\,\pi} \right)^2 = 54{,}600{,}860 \text{ lb-in.}^2$$

f. **Fuselage Yawing I_o**

$$I_{oz} = I_{oy} = 1{,}422{,}807{,}855 \text{ lb-in.}^2$$

g. **Horizontal Stabilizer Pitching I_o**

$$C_a = \frac{b_H \tan \Lambda_{LE_H}}{2} = \frac{(400)\tan 12°}{2} = 43 \text{ in.}$$

$$C_b = c_{t_H} + \frac{b_H \tan \Lambda_{LE_H}}{2} = 50 + 43 = 93 \text{ in.}$$

$$C_c = c_{r_H} = 100 \text{ in.}$$

(equation 8.1-*l*)

$K_a = 0.771$ (constant for any tail surface)

$$\rho = \frac{W_H}{.5(-C_a + C_b + C_c)} \quad \text{(equation 8.1-h)}$$

$$= \frac{1000}{.5(-43 + 93 + 100)} = 13.3 \text{ lb/in.}$$

$$Wx = \frac{\rho}{6}[-C_a^2 + C_b^2 + C_cC_b + C_c^2] \quad \text{(equation 8.1-i)}$$

$$W_Hx = 2.2[-(43)^2 + (93)^2 + (100)(93) + (100)^2] = 57{,}420 \text{ lb-in.}$$

$$I = \frac{\rho}{12}[-C_a^3 + C_b^3 + C_c^2C_b + C_cC_b^2 + C_c^3] \quad \text{(equation 8.1-j)}$$

$$= 1.1[-(43)^3 + (93)^3 + (100)^2(93) + (100)(93)^2 + (100)^3] = 3{,}871{,}725 \text{ lb-in.}^2$$

$$I_{oy} = K_a\left[I - \frac{W_Hx^2}{W_H}\right] \quad \text{(equation 8.1-k)}$$

$$= (0.771)\left[3{,}871{,}725 - \frac{(57{,}420)^2}{1000}\right] = 443{,}070 \text{ lb-in.}^2$$

h. **Horizontal Stabilizer Rolling I_o**

$$\frac{\bar{y}_H}{\frac{b_H}{6}\left(\frac{c_{r_H} + 2c_{t_H}}{c_{r_H} + c_{t_H}}\right)} = \frac{80}{\frac{400}{6}\left(\frac{100 + 100}{100 + 50}\right)} = 0.90$$

$K_4 = 0.740$ (figure 8.1-26)

$$I_{ox} = \frac{W_H b_H^2 K_4}{24}\left(\frac{c_{r_H} + 3c_{t_H}}{c_{r_H} + c_{t_H}}\right) \quad \text{(equation 8.1-r)}$$

$$= \frac{(1000)(400)^2(0.74)}{24}\left(\frac{100 + 150}{100 + 50}\right) = 8{,}222{,}222 \text{ lb-in.}^2$$

i. **Horizontal Stabilizer Yawing I_o**

$$I_{oz} = I_{oy} + I_{ox} \quad \text{(equation 8.1-s)}$$

$$= 443{,}070 + 8{,}222{,}222 = 8{,}665{,}292 \text{ lb-in.}^2$$

j. **Vertical Stabilizer Rolling I_o**

$$\frac{\bar{z}_V}{\frac{b_{V_1}}{3}\left(\frac{c_{r_V} + 2c_{t_V}}{c_{r_V} + c_{t_V}}\right)} = \frac{75}{\frac{200}{3}\left(\frac{250 + 200}{250 + 100}\right)} = 0.87$$

$K_5 = 0.930$ (figure 8.1-27)

$$I_{ox} = \frac{W_V b_{V_1}^2 K_5}{18}\left[1 + \frac{2c_{r_V}c_{t_V}}{(c_{t_V} + c_{t_V})^2}\right] \quad \text{(equation 8.1-t)}$$

$$= \frac{(300)(200)^2(0.93)}{18}\left[1 + \frac{2(250)(100)}{(250+100)^2}\right] = 872{,}960 \text{ lb-in.}^2$$

k. **Vertical Stabilizer Yawing I_o**

$C_a = b_{V_1} \tan\Lambda_{LE_V} = (200) \tan 37° = 150$ in.

$C_b = c_{t_V} + b_{V_1} \tan\Lambda_{LE_V} = 100 + 150 = 250$ in.

$C_c = c_{t_V} = 250$ in.

(equation 8.1-ℓ)

$K_o = 0.771$ (constant for any tail surface)

$$\rho = \frac{W_V}{.5(-C_a + C_b + C_c)} \quad \text{(equation 8.1-h)}$$

$$= \frac{300}{.5(-150 + 250 + 250)} = 1.7 \text{ lb/in.}$$

$$W_V x = \frac{\rho}{6}[-C_a^2 + C_b^2 + C_cC_b + C_c^2] \quad \text{(equation 8.1-i)}$$

$$= 0.28[-(150)^2 + (250)^2 + (250)(250) + (250)^2] = 46{,}200 \text{ lb-in.}$$

$$I = \frac{\rho}{12}[-C_a^3 + C_b^3 + C_c^2C_b + C_cC_b^2 + C_c^3] \quad \text{(equation 8.1-j)}$$

$$= 0.14[-(150)^3 + (250)^3 + (250)^2(250) + (250)(250)^2 + (250)^3] = 8{,}277{,}500 \text{ lb-in.}^2$$

$$I_{oz} = K_o\left[I - \frac{(W_V x)^2}{W_V}\right] \quad \text{(equation 8.1-k)}$$

$$= (0.771)\left[8{,}277{,}500 - \frac{(46{,}200)^2}{300}\right] = 896{,}442 \text{ lb-in.}^2$$

l. **Vertical Stabilizer Pitching I_o**

$$I_{oy} = I_{ox} + I_{oz} \quad \text{(equation 8.1-u)}$$

$$= 872{,}960 + 896{,}442 = 1{,}769{,}402 \text{ lb-in.}^2$$

m. **Power Plant Pitching I_o**

$$I_{oy} = 0.061\left[\frac{3}{4} W_p d_e^2 + W_e \ell_e^2 + (W_p - W_e)\,\ell_p^2\right] \quad \text{(equation 8.1-v)}$$

$$= 0.061\left[\frac{3}{4}(10{,}000)(50)^2 + (7000)(100)^2 + (10{,}000 - 7000)(200)^2\right]$$

$$= 12{,}733{,}750 \text{ lb-in.}^2$$

n. **Power Plant Rolling I_o**

$$I_{ox} = 0.083\, W_p d_e^2 \quad \text{(equation 8.1-w)}$$

$$= (0.083)(10{,}000)(50)^2 = 2{,}075{,}000 \text{ lb-in.}^2$$

o. **Power Plant Yawing I_o**

$$I_{oz} = I_{oy} = 12{,}733{,}750 \text{ lb-in.}^2 \quad \text{(equation 8.1-x)}$$

3. Determine the I_o values for the expendable and variable loads items

a. Fuel I_o

Estimate fuel volume as a rectangular flat plate

Span = 600 in. Chord = 150 in. Thickness = 8 in.

Using the conventional inertia formula for a rectangular parallelopiped

$$I_{oy} = \frac{W}{12}(c^2 + t^2) = \frac{20{,}000}{12}[(150)^2 + (8)^2] = 37{,}606{,}667 \text{ lb-in.}^2$$

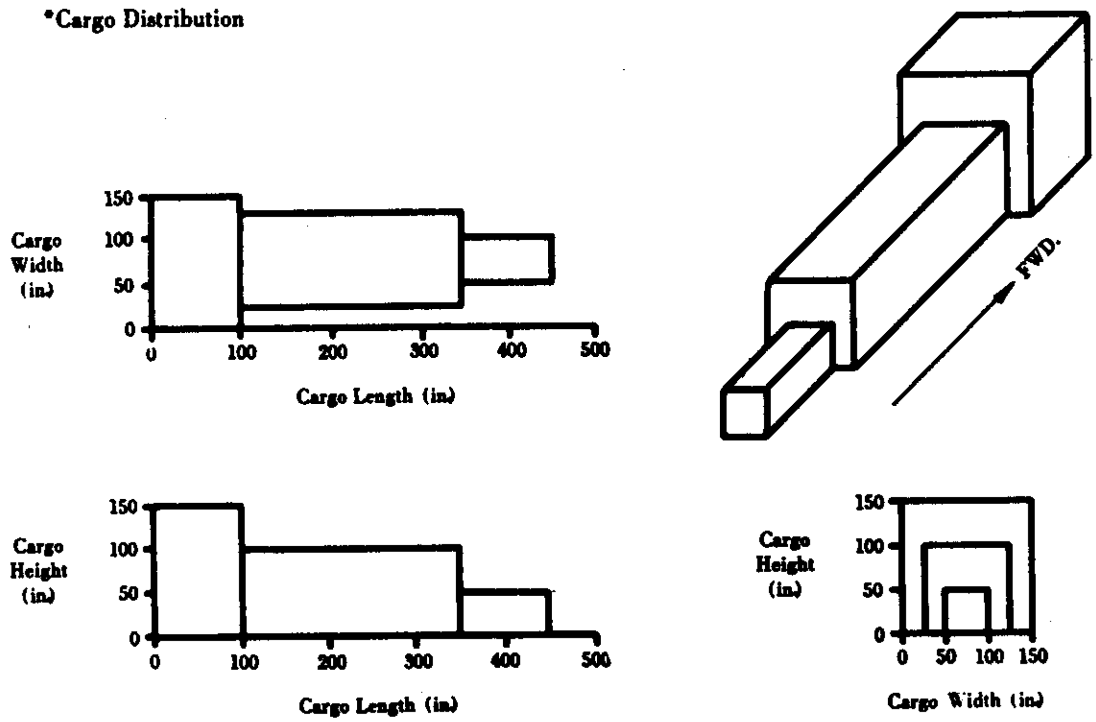
$$I_{ox} = \frac{W}{12}(b^2 + t^2) = \frac{20{,}000}{12}[(600)^2 + (8)^2] = 600{,}106{,}667 \text{ lb-in.}^2$$

$$I_{oz} = \frac{W}{12}(c^2 + b^2) = \frac{20{,}000}{12}[(150)^2 + (600)^2] = 637{,}500{,}000 \text{ lb-in.}^2$$

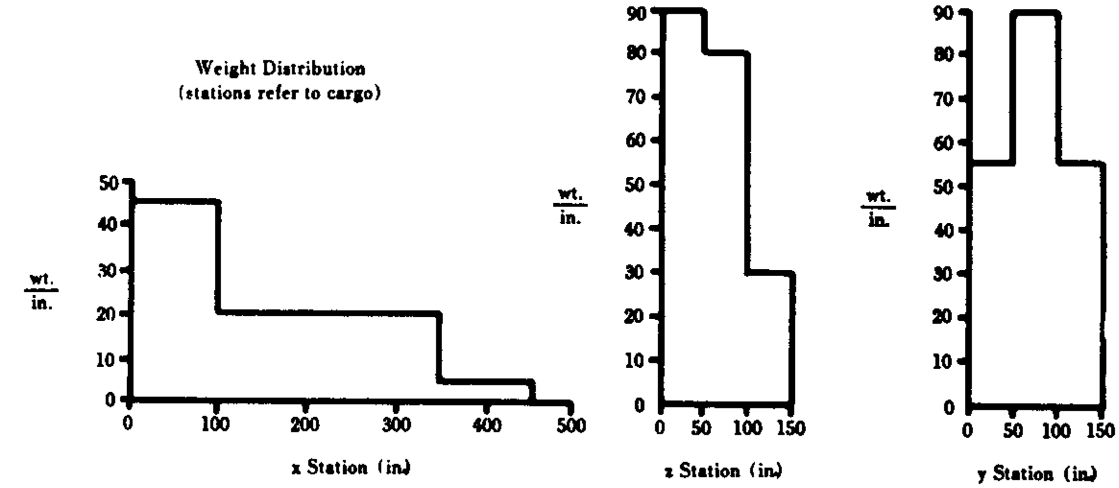
where W is the fuel weight

b. Cargo I_o

*Cargo Distribution



*Indicated as a sample only to show an approach to cargo inertia determinations, and may not reflect actual cargo distributions.



Centroids:*

$$\bar{x} = \frac{\int_0^{100} 45\,x\,dx + \int_{100}^{350} 20\,x\,dx + \int_{350}^{450} 5\,x\,dx}{10{,}000} = 155.0$$

$$\bar{z} = \frac{\int_0^{50} 90\,z\,dz + \int_{50}^{100} 80\,z\,dz + \int_{100}^{150} 30\,z\,dz}{10{,}000} = 60.0$$

$$\bar{y} = \frac{\int_0^{50} 55\,y\,dy + \int_{50}^{100} 90\,y\,dy + \int_{100}^{150} 55\,y\,dy}{10{,}000} = 75.0$$

Second Moments:*

$$Wx^2 = \int_0^{100} 45\,x^2dx + \int_{100}^{350} 20\,x^2dx + \int_{350}^{450} 5\,x^2dx = 374{,}583{,}333$$

$$Wz^2 = \int_0^{50} 90\,z^2dz + \int_{50}^{100} 80\,z^2dz + \int_{100}^{150} 30\,z^2dz = 50{,}833{,}333$$

$$Wy^2 = \int_0^{50} 55\,y^2dy + \int_{50}^{100} 90\,y^2dy + \int_{100}^{150} 55\,y^2dy = 72{,}083{,}333$$

$I_{oy} = I_y - W\bar{r}^2$ (equation 8.1-f)

$= Wx^2 - W\bar{x}^2 + Wz^2 - W\bar{z}^2$ (Substituting equation 8.1-c in 8.1-f)

$= 374{,}583{,}333 - (10{,}000)(155)^2 + 50{,}833{,}333 - (10{,}000)(60)^2$

$= 149{,}166{,}666$ lb-in.^{2}

* The x, y, and z distances, as well as the Wx^2, Wz^2, and Wy^2 terms in the equations for Centroids and Second Moments, refer only to these calculations and are not to be confused with the distances and moment terms in Table 8.1-A.

$$I_{oz} = I_x - W\bar{r}^2 \quad \text{(equation 8.1-f)}$$

$$= Wz^2 - W\bar{z}^2 + Wy^2 - W\bar{y}^2$$

$$\doteq 50{,}833{,}334 - (10{,}000)\,(60)^2 + 72{,}083{,}333 - (10{,}000)\,(75)^2$$

$$= 30{,}666{,}666 \text{ lb-in.}^2$$

$$I_{oz} = I_z - W\bar{r}^2 \quad \text{(equation 8.1-f)}$$

$$= Wy^2 - W\bar{y}^2 + Wx^2 - W\bar{x}^2$$

$$= 72{,}083{,}333 - (10{,}000)\,(75)^2 + 374{,}583{,}333 - (10{,}000)\,(155)^2$$

$$= 150{,}166{,}666 \text{ lb-in.}^2$$

4. Determine the total airplane inertias in pitch, roll, and yaw. All of the values calculated in 1, 2, and 3 are tabulated in Table 8.1-A and each column is totaled.

a. Inertia about the remote axis

$$I_y = \sum (Wr^2 + I_{oy}) \quad \text{(equation 8.1-e)}$$

$$= \sum [W(x^2 + z^2) + I_{oy}]$$

$$= (28{,}946{,}000 + 2{,}279{,}500)\,10^3 + (1{,}688{,}505)\,10^3$$

$$= 32{,}914{,}005 \times 10^3 \text{ lb-in.}^2$$

$$I_x = \sum (Wr^2 + I_{ox}) \quad \text{(equation 8.1-e)}$$

$$= \sum [W(y^2 + z^2) + I_{ox}]$$

$$= (400{,}000 + 2{,}279{,}500)\,10^3 + (1{,}324{,}670)\,10^3$$

$$= 4{,}004{,}170 \times 10^3 \text{ lb-in.}^2$$

$$I_z = \sum (Wr^2 + I_{oz}) \quad \text{(equation 8.1-e)}$$

$$= \sum [W(y^2 + x^2) + I_{oz}]$$

$$= (400{,}000 + 28{,}946{,}000)\,10^3 + (2{,}924{,}872)\,10^3$$

$$= 32{,}270{,}872 \times 10^3 \text{ lb-in.}^2$$

b. Inertia about the airplane centroid

Pitching $I_o = I_y - W\bar{r}^2$ (equation 8.1-f)

where $\bar{r} = \dfrac{\sum Wr}{\sum W}$ (equation 8.1-g)

$$I_o = I_y - \left[\frac{\sum (Wx)^2 + \sum (Wz)^2}{\sum W}\right]$$

$$= 32{,}914{,}005 \times 10^3 - \left[\frac{(46{,}460{,}000)^2 + (13{,}040{,}000)^2}{76{,}300}\right]$$

$$= 2{,}395{,}352 \times 10^3 \text{ lb-in.}^2$$

Rolling $I_o = I_x - W\bar{r}^2$ (equation 8.1-f)

$$= I_x - \left[\frac{\sum (Wy)^2 + \sum (Wz)^2}{\sum W}\right]$$

$$= 4{,}004{,}170 \times 10^3 - \frac{(2{,}000{,}000)^2 + (13{,}040{,}000)^2}{76{,}300}$$

$= 1,723,153 \times 10^3$ lb-in.²

Yawing $I_o = I_z - W\bar{r}^2$ (equation 8.1-f)

$$= I_z - \left[\frac{\sum (Wx)^2 + \sum (Wy)^2}{\sum W}\right]$$

$$= 32,270,872 \times 10^3 - \frac{(46,460,000)^2 + (2,000,000)^2}{76,300}$$

$= 3,928,387 \times 10^3$ lb-in.²

See Table 8.1-A for tabulated results and conversion to units of mass times distance squared.

TABLE. 8.1-A

SECTION	WEIGHT (lbs)	x (in.)	z (in.)	y (in.)	Wx × 10⁻³ (lb-in.)	Wz × 10⁻³ (lb-in.)	Wy × 10⁻³ (lb-in.)	Wx² × 10⁻³ (lb-in.²)	Wz² × 10⁻³ (lb-in.²)	Wy² × 10⁻³ (lb-in.²)
Wing	15,000	650	150		9750	2250		6,337,500	337,500	
Fuselage	20,000	600	200		12,000	4000		7,200,000	800,000	
H. Stab.	1000	1150	200		1150	200		1,322,500	40,000	
V. Stab.	300	1200	300		360	90		432,000	27,000	
P. Plant	10,000	520	150	200	5200	1500	2000	2,704,000	225,000	400,000
Fuel	20,000	650	150		13,000	3000		8,450,000	450,000	
Cargo	10,000	500	200		5000	2000		2,500,000	400,000	
Subtotal	76,300	608.91	170.90	—	46,460	13,040	2000	28,946,000	2,279,500	400,000

SECTION	PITCH $I_{oy} \times 10^{-3}$ (lb-in.²)	ROLL $I_{ox} \times 10^{-3}$ (lb-in.²)	YAW $I_{oz} \times 10^{-3}$ (lb-in.²)
Wing	43,977	628,125	672,102
Fuselage	1,442,808	54,601	1,442,808
H. Stab.	443	8222	8665
V. Stab.	1769	873	896
P. Plant	12,734	2075	12,734
Fuel	37,607	600,107	637,500
Cargo	149,167	30,667	150,167
Subtotal	1,688,505	1,324,670	2,924,872
Total I_o ~ lb-in.²	2,395,352	1,723,153	3,928,397
Total I_o ~ slug-ft²*	517	372	848

*For conversion to slug-ft², multiply lb-in.² by $\left(\frac{1 \text{ ft}^2}{144 \text{ in.}^2}\right)\left(\frac{1}{32.17 \text{ ft/sec}^2}\right)$ $\left(\frac{1 \text{ ft}^2}{4632.48 \text{ in.}^2\text{-ft/sec}^2}\right)$

This conversion factor gives an inertia value at sea level, since the value of 32.17 ft/sec² is the standard acceleration of gravity at sea level.

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2. Society of Aeronautical Weight Engineers, Texas Chapter: An Introduction to Aeronautical Weight Engineering. First Edition, Chance Vought Aircraft, Inc., Dallas, Texas; Chapter 21, 1959. (U)

3. Anon: S.A.W.E. Weight Handbook, Volume 1: Third Edition, Society of Aeronautical Weight Engineers, Inc., 1944. (U)

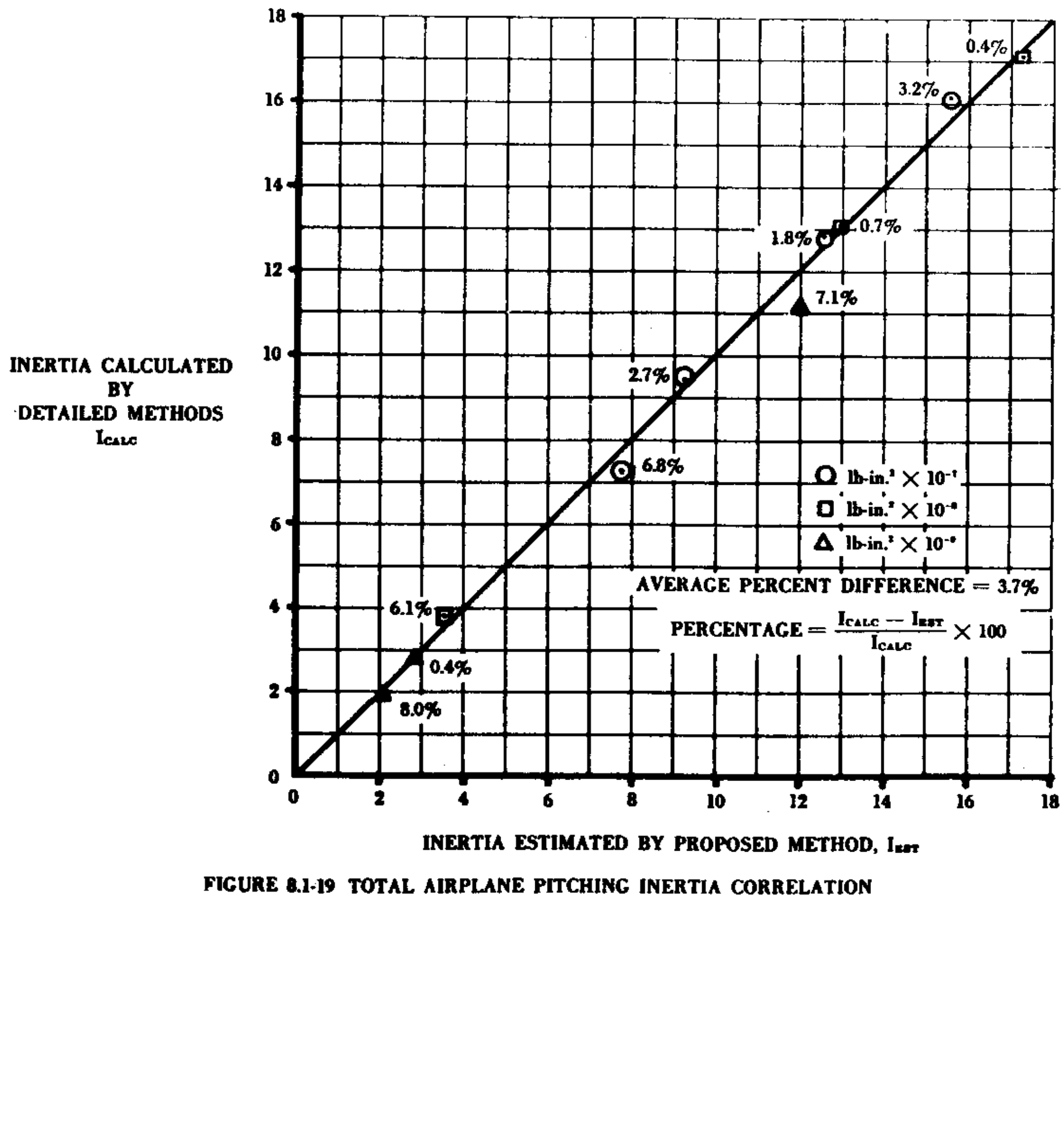


FIGURE 8.1-19 TOTAL AIRPLANE PITCHING INERTIA CORRELATION

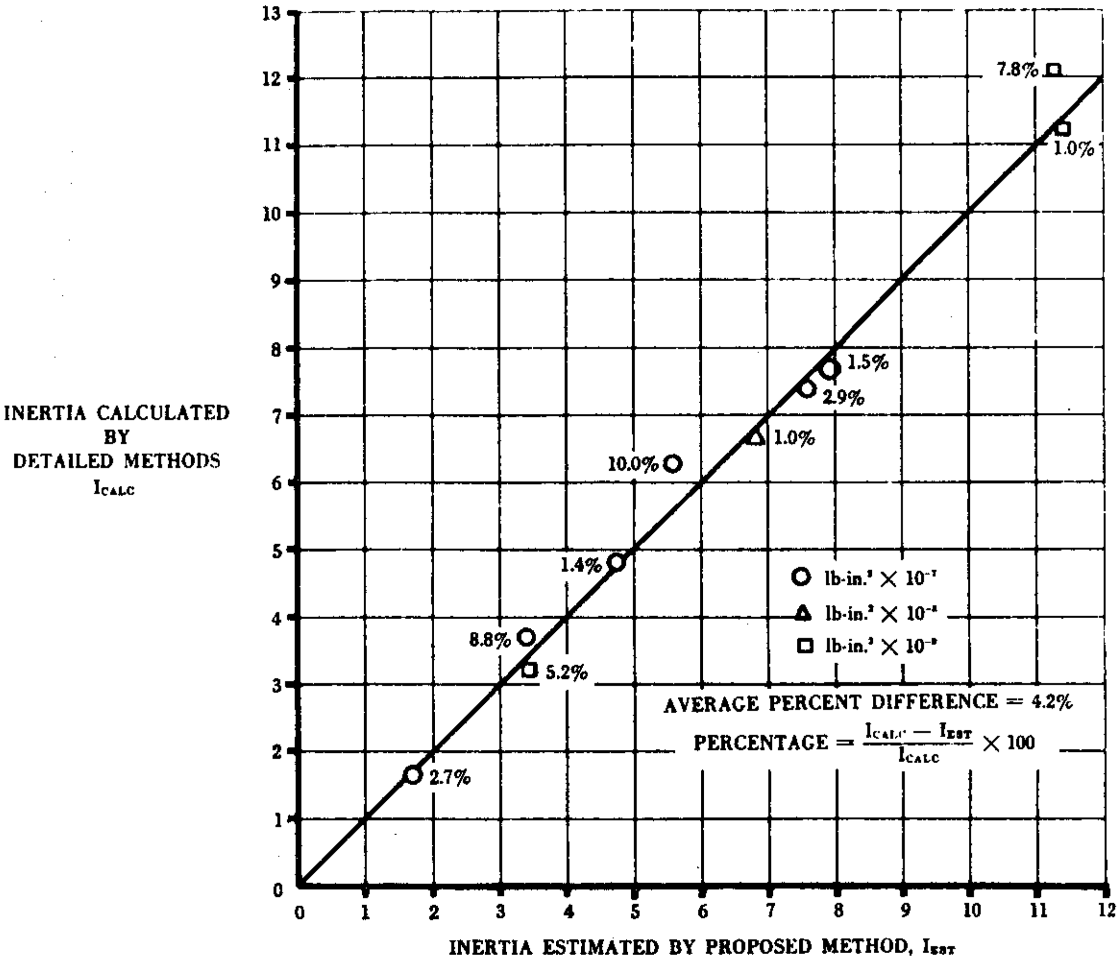


FIGURE 8.1-20 TOTAL AIRPLANE ROLLING INERTIA CORRELATION

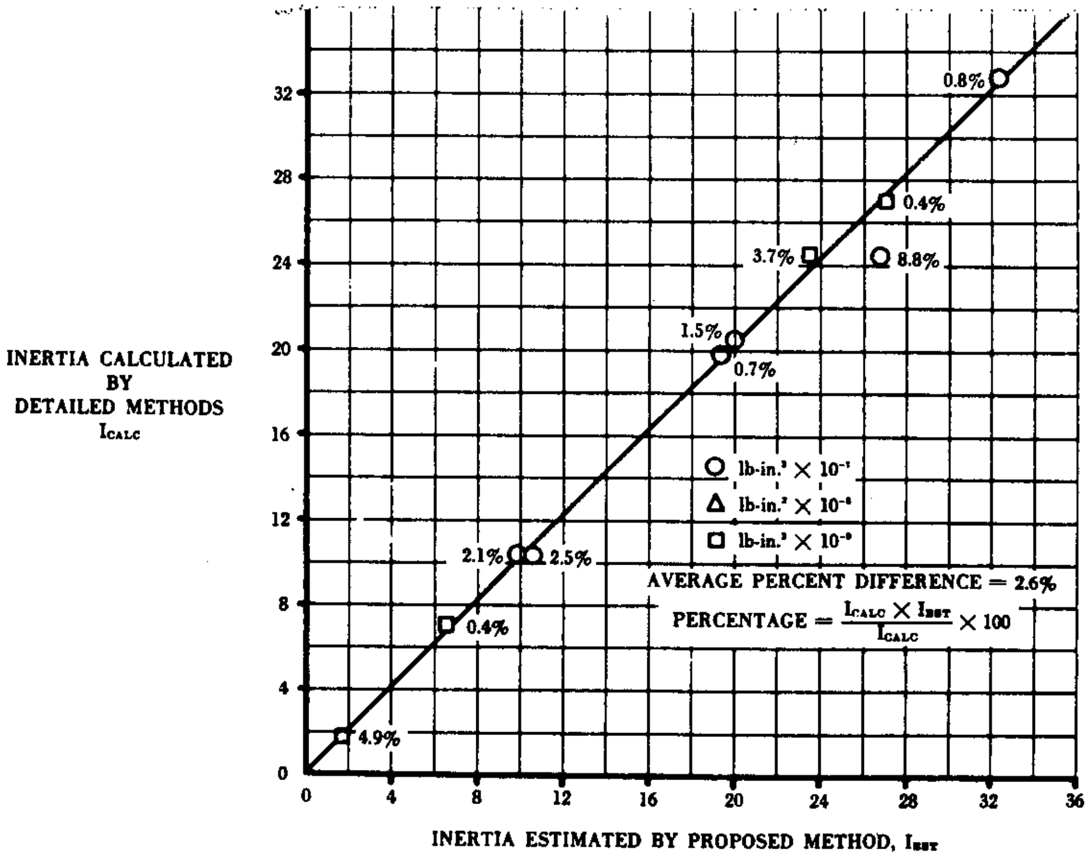
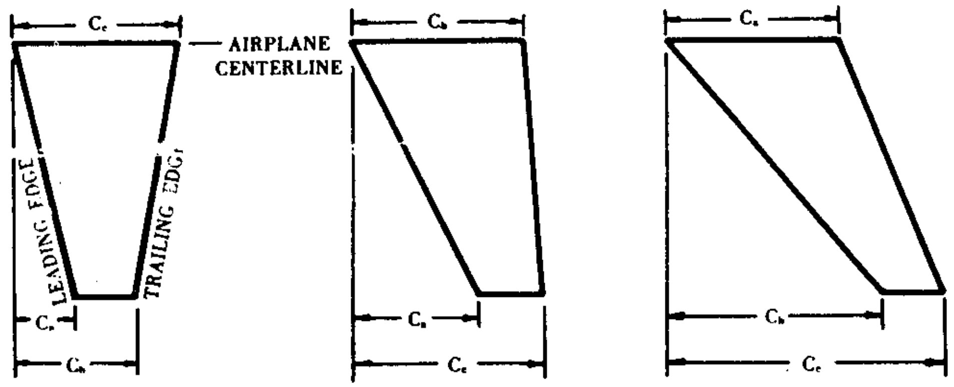
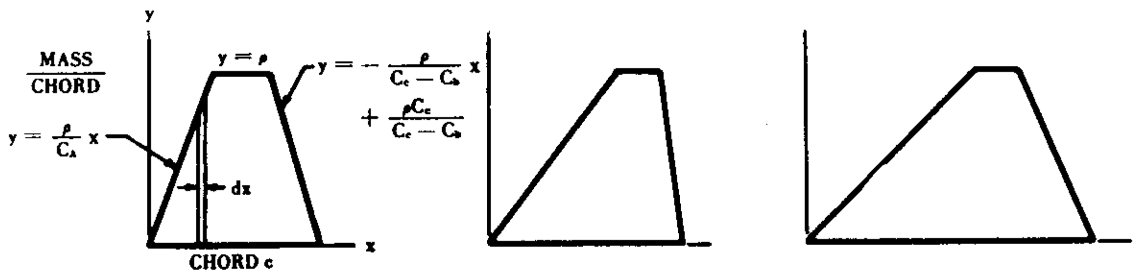


FIGURE 8.1-21 TOTAL AIRPLANE YAWING INERTIA CORRELATION



WING SHAPES



MASS DISTRIBUTIONS

$$\sum m = \int_0^{C_a} \frac{\rho}{C_a} x\,dx + \int_{C_a}^{C_b} \rho\,dx - \int_{C_b}^{C_c} \frac{\rho}{C_c - C_b} x\,dx + \int_{C_b}^{C_c} \frac{\rho C_c}{C_c - C_b} dx = \frac{\rho(-C_a + C_b + C_c)}{2}$$

$$\sum mx = \int_0^{C_a} \frac{\rho}{C_a} x^2 dx + \int_{C_a}^{C_b} \rho\,x\,dx - \int_{C_b}^{C_c} \frac{\rho}{C_c - C_b} x^2 dx + \int_{C_b}^{C_c} \frac{\rho C_c}{C_c - C_b} x\,dx = \frac{\rho(-C_a^2 + C_b^2 + C_c C_b + C_c^2)}{6}$$

$$I = \sum mx^2 = \int_0^{C_a} \frac{\rho}{C_a} x^3 dx + \int_{C_a}^{C_b} \rho\,x^2 dx - \int_{C_b}^{C_c} \frac{\rho}{C_c - C_b} x^3 dx + \int_{C_b}^{C_c} \frac{\rho C_c}{C_c - C_b} x^2 dx$$

$$= \frac{\rho(-C_a^3 + C_b^3 + C_c^2 C_b + C_c C_b^2 + C_c^3)}{12}$$

$$I_{oy} = K_o\left[I - \frac{(\sum mx)^2}{\sum m}\right]$$

where

$K_o = 0.703$ for any wing

$K_o = 0.771$ for any horizontal or vertical stabilizer

FIGURE 8.1-22 WING PITCHING INERTIA CONSIDERATIONS

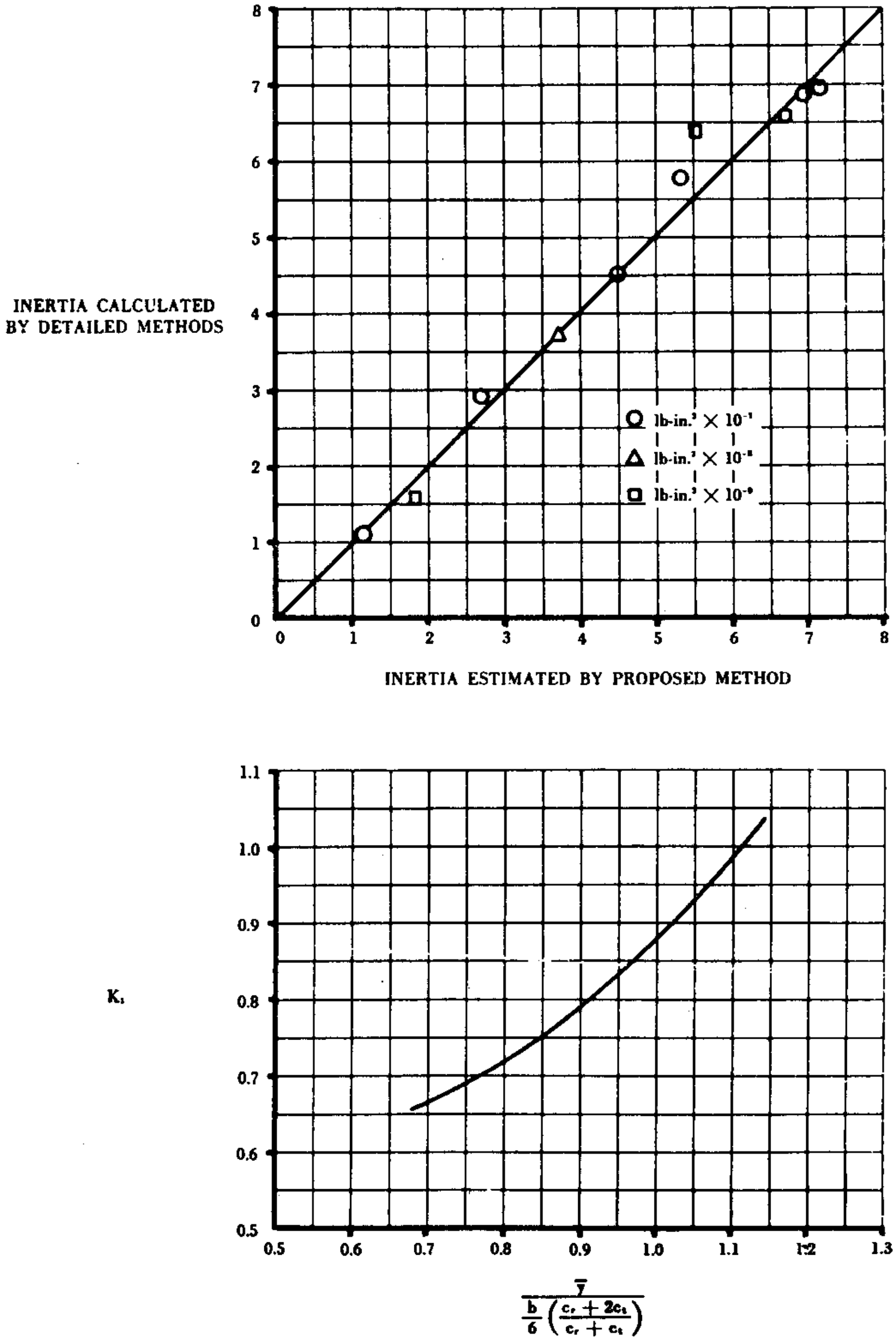


FIGURE 8.1-23 WING ROLLING I_x CORRELATION AND K FACTOR

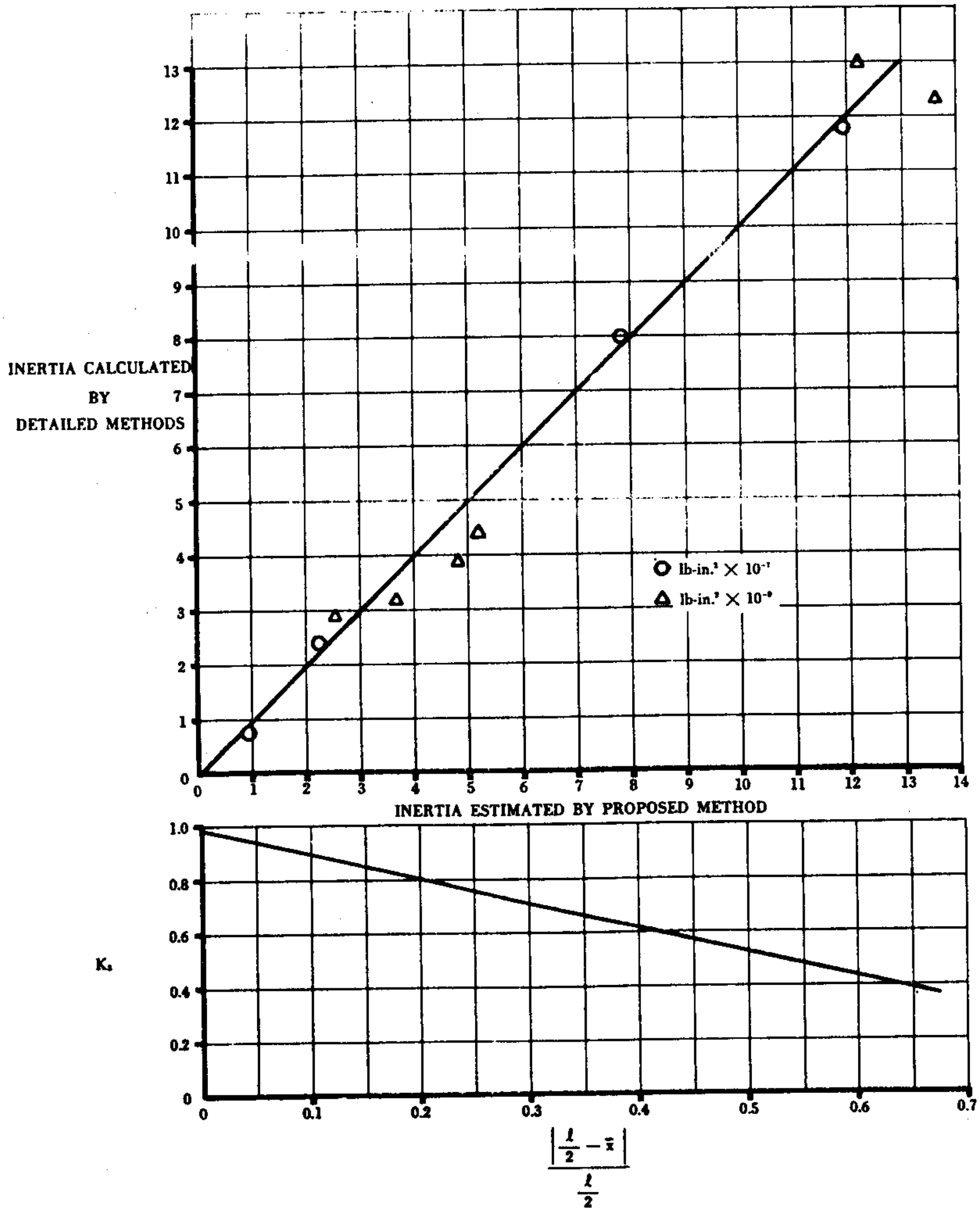


FIGURE 8.1-24 FUSELAGE PITCHING I, CORRELATION AND K FACTOR

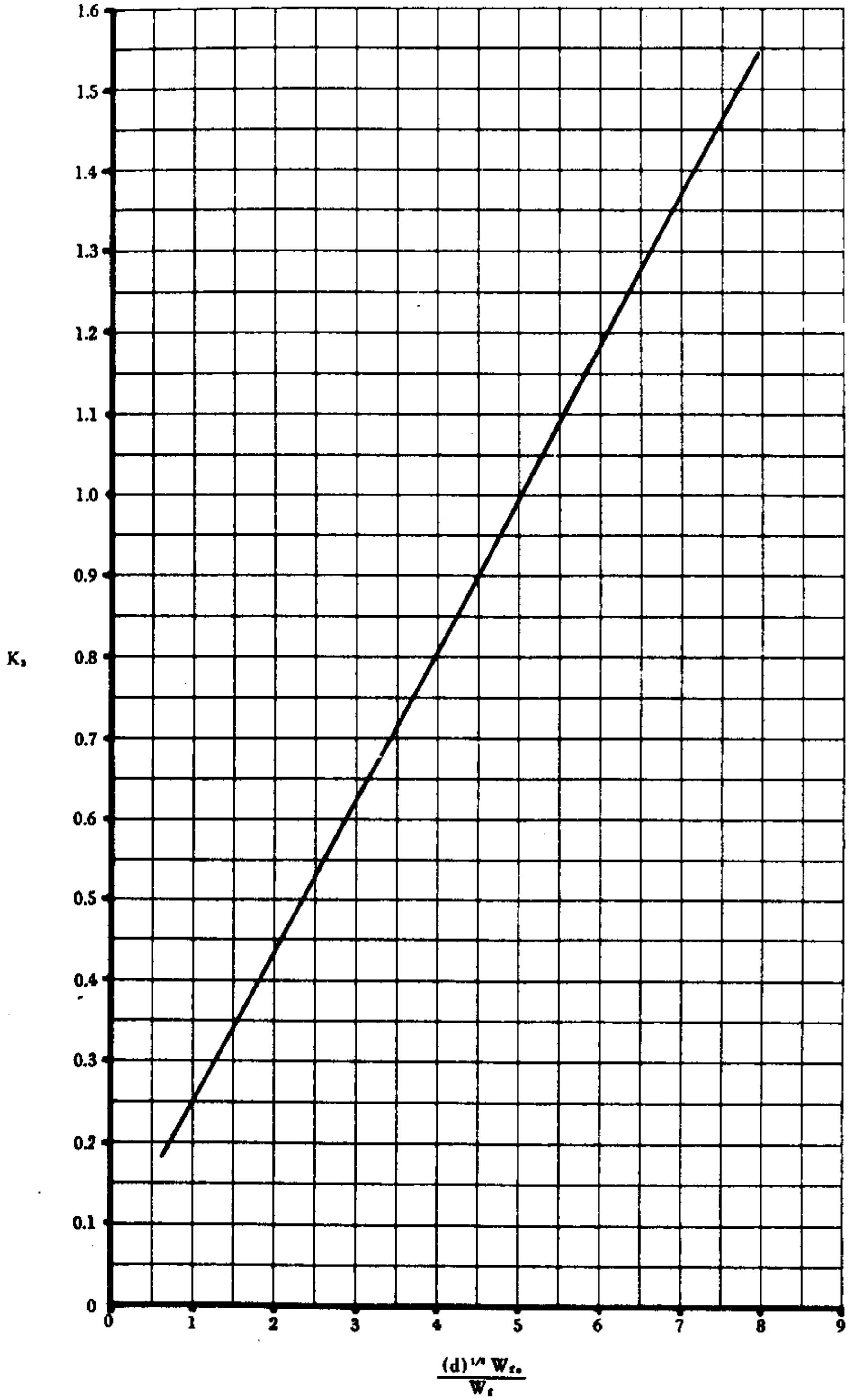
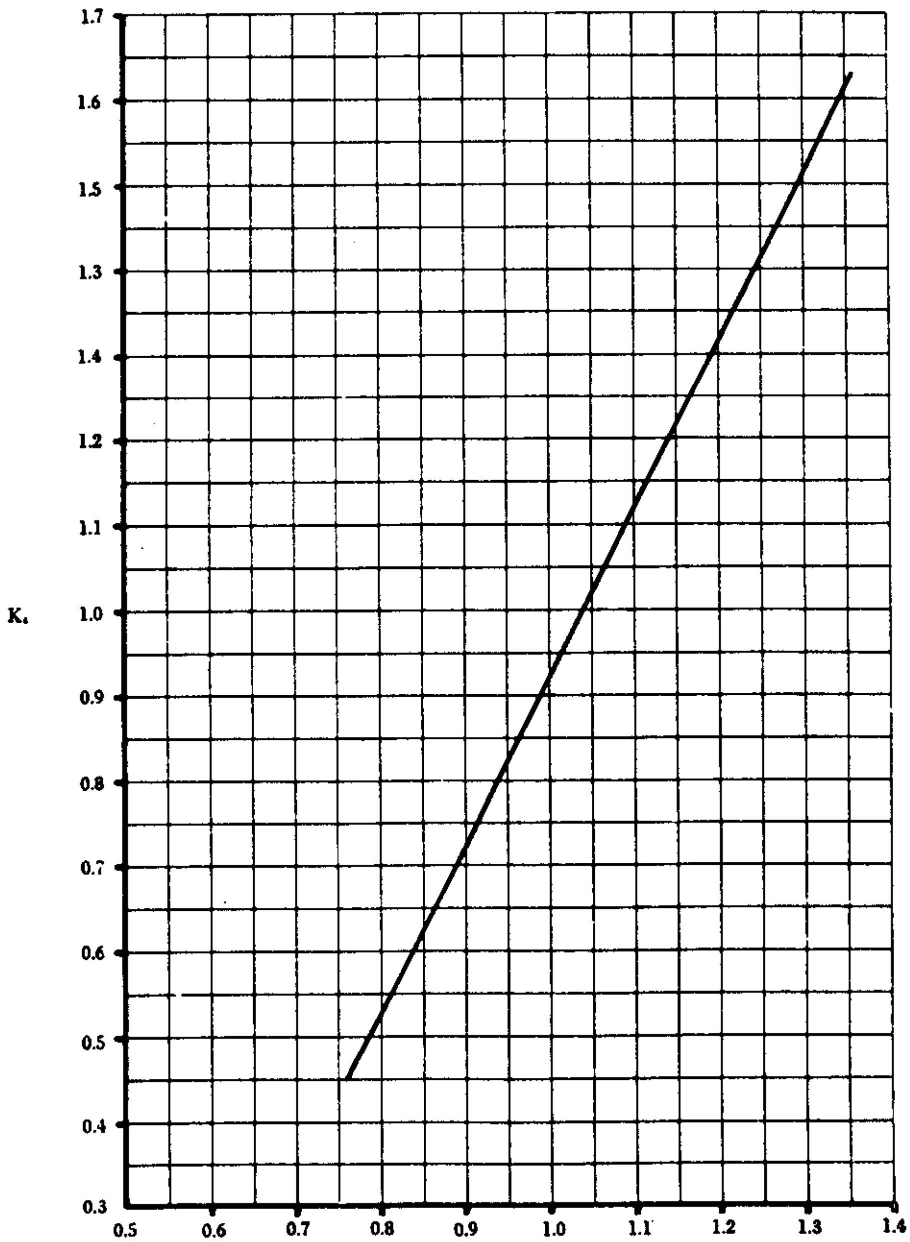
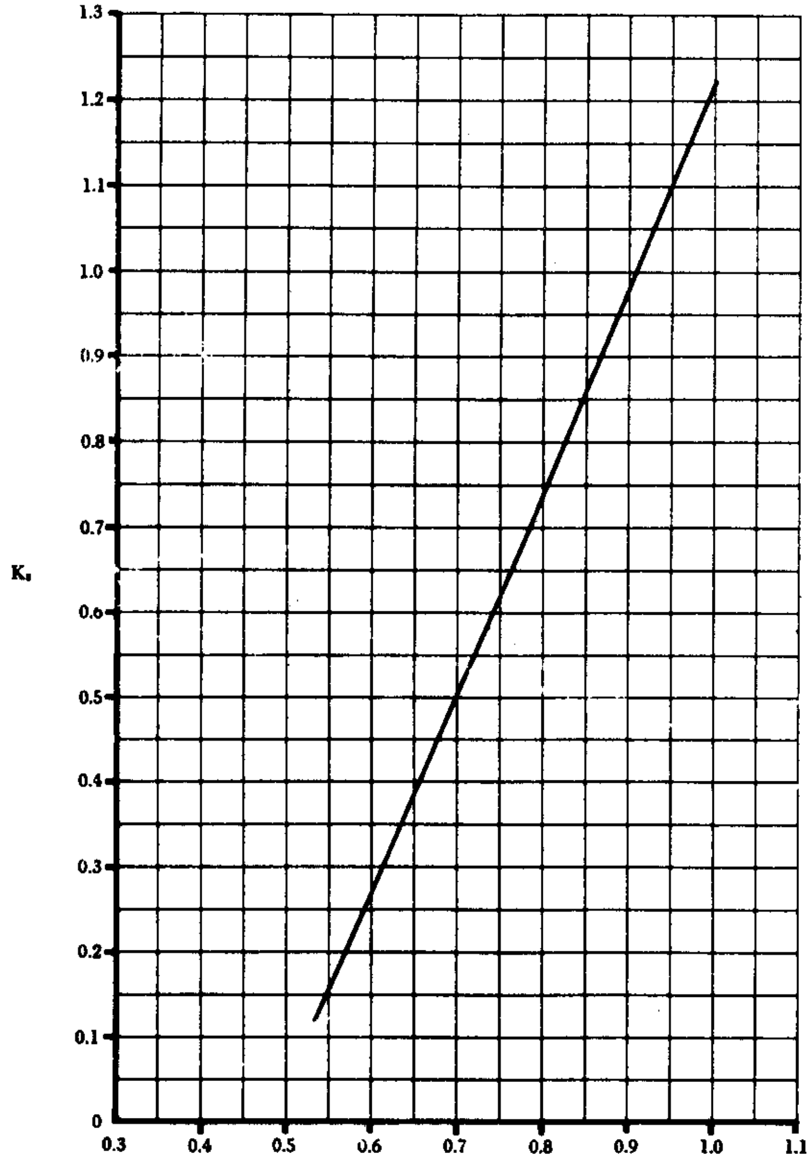


FIGURE 8.1-25 FUSELAGE ROLLING I. K FACTOR



$$\frac{\bar{y}_H}{\frac{b_H}{6}\left(\frac{c_{H_r} + 2c_{H_t}}{c_{H_r} + c_{H_t}}\right)}$$

FIGURE 8.1-26 HORIZONTAL STABILIZER ROLLING I, K FACTOR



$$\frac{\bar{z}_V}{\frac{b_V}{3}\left(\frac{c_{V_r} + 2c_{V_t}}{c_{V_r} + c_{V_t}}\right)}$$

FIGURE 8.1-27 VERTICAL STABILIZER ROLLING I. K FACTOR

8.2 MISSILE MASS AND INERTIA

Two methods of estimating missile mass and inertia, each limited by the initial assumptions and input data, are presented in this section. The method selected will depend on the desired accuracy related to the problem at hand.

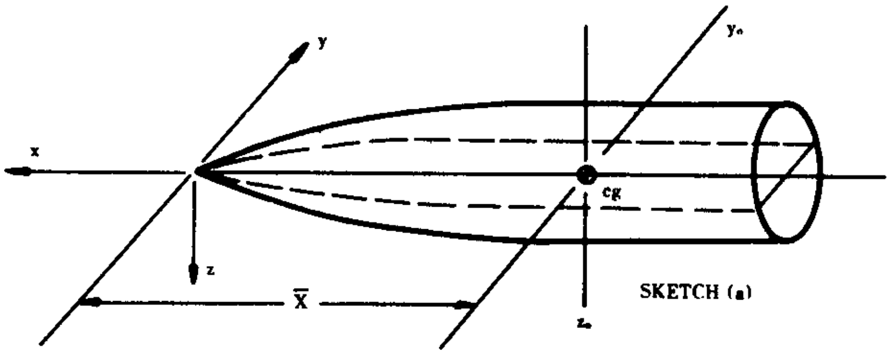
The first method, more sophisticated than the second, requires an estimated weight and center-of-gravity breakdown of the vehicle. Although the results are presented in the Datcom for hand calculations, they are most expeditiously programed for a small-capacity computer. The hand computations may be long and tedious, but they are not complicated.

The second method presents the procedure, based on estimates of gross stage weight and profile, for determining the order of magnitude of pitch and roll moments of inertia of a vehicle.

DATCOM METHODS

Method 1

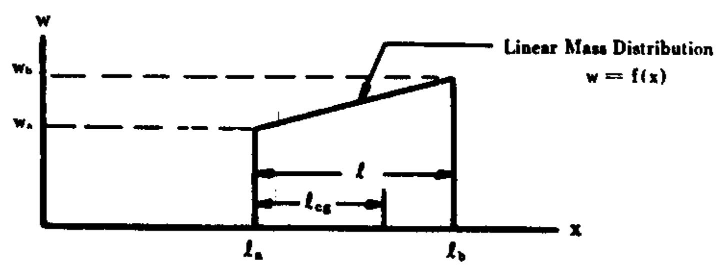
This method assumes that the basic body being analyzed is symmetrical about one axis, as illustrated in the axis-system diagram of sketch (a). This leads to the assumption that the center of gravity of the body lies on the longitudinal axis of the vehicle. However, this assumption does not eliminate handling of missile components that are not symmetrical.



SKETCH (a)

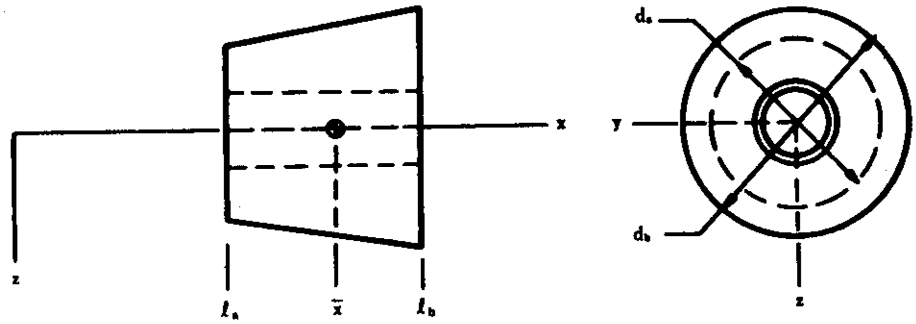
A second assumption is that the moment of inertia with respect to the z_o axis is equal to the moment of inertia with respect to the y_o axis. (yaw moment of inertia = pitch moment of inertia). This assumption is reasonable for most missile bodies and will be in the range of accuracy desired for moment-of-inertia values of vehicles in the preliminary-design phase.

A third assumption is that all components can be approximated by a linear mass distribution along the longitudinal axis as shown in sketch (b). This assumption forces the following limitation: $\ell/3 \leq \ell_{c.g.} \leq 2\ell/3$, where ℓ is the length of the component and ℓ_{cg} is the center of gravity of the component measured from the beginning station of the component. The weight per unit length of element is denoted by w.



SKETCH (b)

A fourth assumption is that the diameters of any component can be approximated by a linear distribution along the longitudinal axis as shown in sketch (c).



SKETCH (c)

It will be assumed that W, $\bar{x}$, ℓ_a, ℓ_b, d_a and d_b are given or can be estimated and tabulated for each symmetrical component of the missile, where W is the weight of a given component and $\bar{x}$ is the longitudinal station of the center of gravity of that component with respect to the missile nose apex.

The values w_a and w_b are then estimated for each component from the equations

$$w_a = \frac{W}{\ell_b - \ell_a}\left[4 - \frac{6(\bar{x} - \ell_a)}{\ell_b - \ell_a}\right] \qquad 8.2\text{-a}$$

$$w_b = \frac{W}{\ell_b - \ell_a}\left[\frac{6(\bar{x} - \ell_a)}{\ell_b - \ell_a} - 2\right] \qquad 8.2\text{-b}$$

The pitch and yaw moments of inertia (I_{yy}' and I_{zz}') of each solid (nonliquid) component can then be estimated from

$$I_{yy}' = I_{zz}' = (\ell_b^4 - \ell_a^4)\left(\frac{m}{4} + \frac{mT^2}{4K}\right) + (\ell_b^3 - \ell_a^3)\left(\frac{b}{3} + \frac{2mTN + bT^2}{3K}\right)$$
$$+ (\ell_b^2 - \ell_a^2)\left(\frac{mN^2 + 2bTN}{2K}\right) + (\ell_b - \ell_a)\frac{bN^2}{K} - W\bar{x}^2 \qquad 8.2\text{-c}$$

where

$$m = \frac{w_b - w_a}{\ell_b - \ell_a}$$

$$T = \frac{d_b - d_a}{\ell_b - \ell_a}$$

$$b = w_a - \ell_a\left(\frac{w_b - w_a}{\ell_b - \ell_a}\right)$$

$$N = d_a - \ell_a\left(\frac{d_b - d_a}{\ell_b - \ell_a}\right)$$

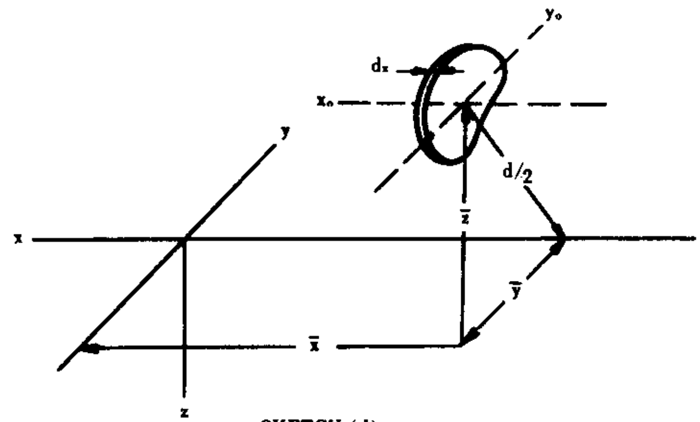
$$K = \frac{16}{1 + \left(\frac{d_o}{d}\right)^2}$$ for elements symmetrical about the longitudinal axis (use figure 8.2-5)

$$= \frac{1}{\left(\frac{z}{d}\right)^2 + \left(\frac{\rho_{y_o}}{d}\right)^2}$$ for unsymmetrical solid elements in pitch (use figure 8.2-6)

$$= \frac{1}{\left(\frac{\bar{y}}{d}\right)^2 + \left(\frac{\rho_{z_o}}{d}\right)^2} \quad \text{for unsymmetrical solid elements in yaw (use figure 8.2-6)}$$

$\bar{x}, \bar{y}, \bar{z}$ are measured to the c.g. of the element as shown in sketch (d)

$\rho_{x_o}, \rho_{y_o}, \rho_{z_o}$ are the radii of gyration of the element



SKETCH (d)

The corresponding roll moment of inertia of a given solid element is

$$I_{xx}' = \frac{2}{C}\left[(\ell_b^4 - \ell_a^4)\frac{mT^2}{4} + (\ell_b^3 - \ell_a^3)\left(\frac{2mTN + bT^2}{3}\right) + (\ell_b^2 - \ell_a^2)\frac{mN^2 + 2bTN}{2} + (\ell_b - \ell_a)\, bN^2\right] \qquad \text{8.2-d}$$

where

$$C = \frac{16}{1 + \left(\frac{d_o}{d}\right)^2} \quad \text{for symmetrical elements (same as K, use figure 8.2-5)}$$

$$= \frac{2}{\frac{1}{4} + \left(\frac{\rho_{x_o}}{d}\right)^2} \quad \text{for unsymmetrical elements (use figure 8.2-7)}$$

and the other symbols are as defined for equation 8.2-c.

For liquid components, the pitch and yaw moments of inertia are

$$I_{yy}' = I_{zz}' = K_L\left[(\ell_b^4 - \ell_a^4)\left(\frac{m}{4} + \frac{mT^2}{64}\right) + (\ell_b^3 - \ell_a^3)\left(\frac{b}{3} + \frac{2mTN + bT^2}{48}\right) + (\ell_b^2 - \ell_a^2)\frac{mN^2 + 2bTN}{32} + (\ell_b - \ell_a)\frac{bN^2}{16} - W\bar{x}^2\right] \qquad \text{8.2-e}$$

where

K_L is given by figure 8.2-8 as a function of the fineness ratio of the element.

The corresponding roll moment of inertia of a liquid element is assumed to be zero.

$$I_{xx}' = 0 \qquad \text{8.2-f}$$

The pitch and yaw moments of inertia of the complete vehicle are obtained by using the parallel-axis theorem to sum the contribution of each element. Thus

$$I_{yy} = I_{zz} = \sum I_{yy}' + \sum W\bar{x}^2 - (\sum W)\bar{X}^2 \qquad 8.2\text{-g}$$

where the sums are taken over the entire set of elements comprising the configuration and $\bar{X}$ is the longitudinal position of the vehicle center of gravity. This latter term is obtained from the equation

$$\bar{X} = \frac{\sum \bar{x} W}{\sum W} \qquad 8.2\text{-h}$$

The corresponding roll moment of inertia for the complete vehicle is

$$I_{xx} = \sum I_{xx}' \qquad 8.2\text{-i}$$

where again the sum extends over all components.

Certain common vehicle components not satisfying the basic assumptions can be handled exactly by replacing them by equivalent shapes that do satisfy the assumptions.

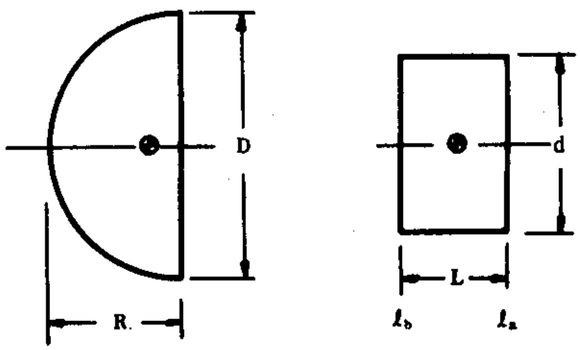
The spherical segment shell, which does not have a linear diameter variation, can be replaced by an equivalent cylindrical shell as illustrated in figure 8.2-9. The moment of inertia of the cylindrical shell can be computed by the derived equations.

The dimensions of an equivalent solid cylinder for a hemispherical solid are given as

$d = 0.894\ D$ $\qquad\qquad$ $l_a = \bar{x} - 0.2105\ D$

$L = 0.421\ D$ $\qquad\qquad$ $l_b = \bar{x} + 0.2105\ D$

where the symbols are defined in sketch (e).



SKETCH (e)

Method 2

For order-of-magnitude determinations, figures 8.2-10 and 8.2-11 summarize the pitch (and yaw) and roll radii of gyration, respectively, of a number of actual missile configurations. This information will allow a determination of these parameters within perhaps ±20 percent. The moments of inertia are then

$$I_{P_{C.G.}} = I_{yy} = I_{zz} = \rho_P^2 \sum W \qquad 8.2\text{-j}$$

$$I_{R_{C.G.}} = I_{xx} = \rho_R^2 \sum W \qquad 8.2\text{-k}$$

where $\sum W$ is the total weight of the vehicle

ρ_P and ρ_R are obtained from figures 8.2-10 and 8.2-11, respectively.

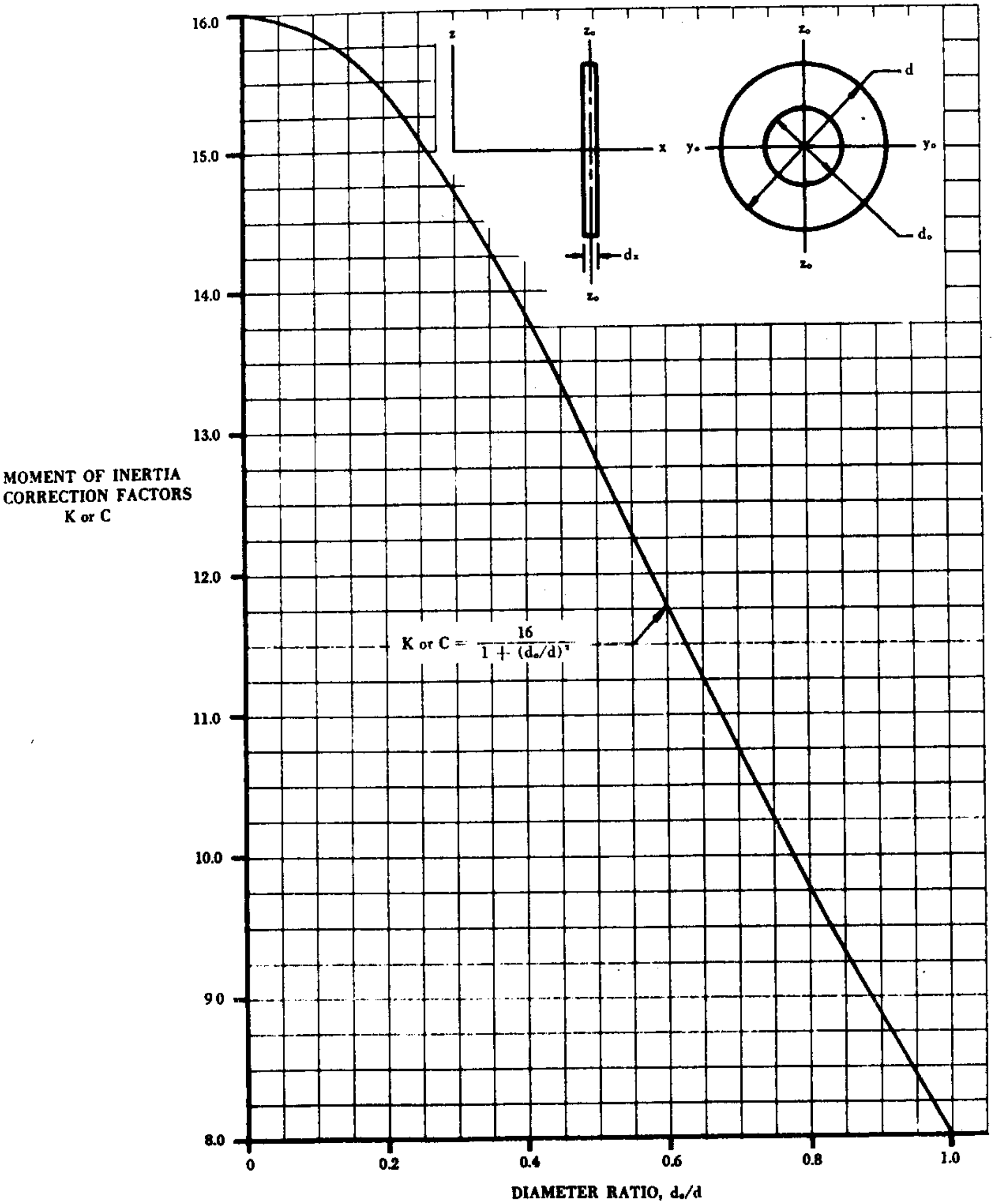


FIGURE 8.2-5 SYMMETRICAL SOLID COMPONENT
MOMENT OF INERTIA CORRECTION FACTORS K AND C

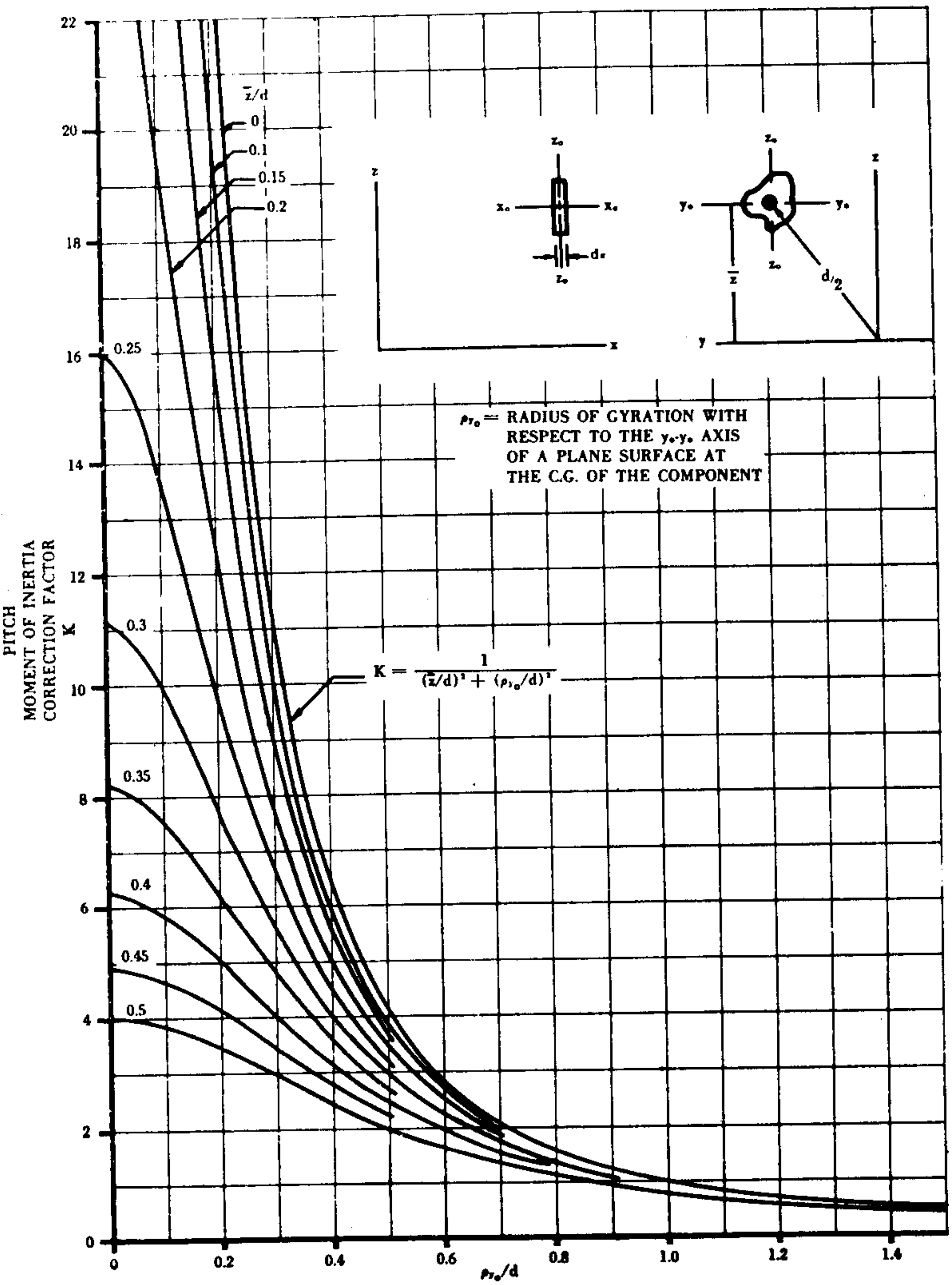


FIGURE 8.2-6 UNSYMMETRICAL SOLID COMPONENT
PITCH MOMENT OF INERTIA CORRECTION FACTOR K

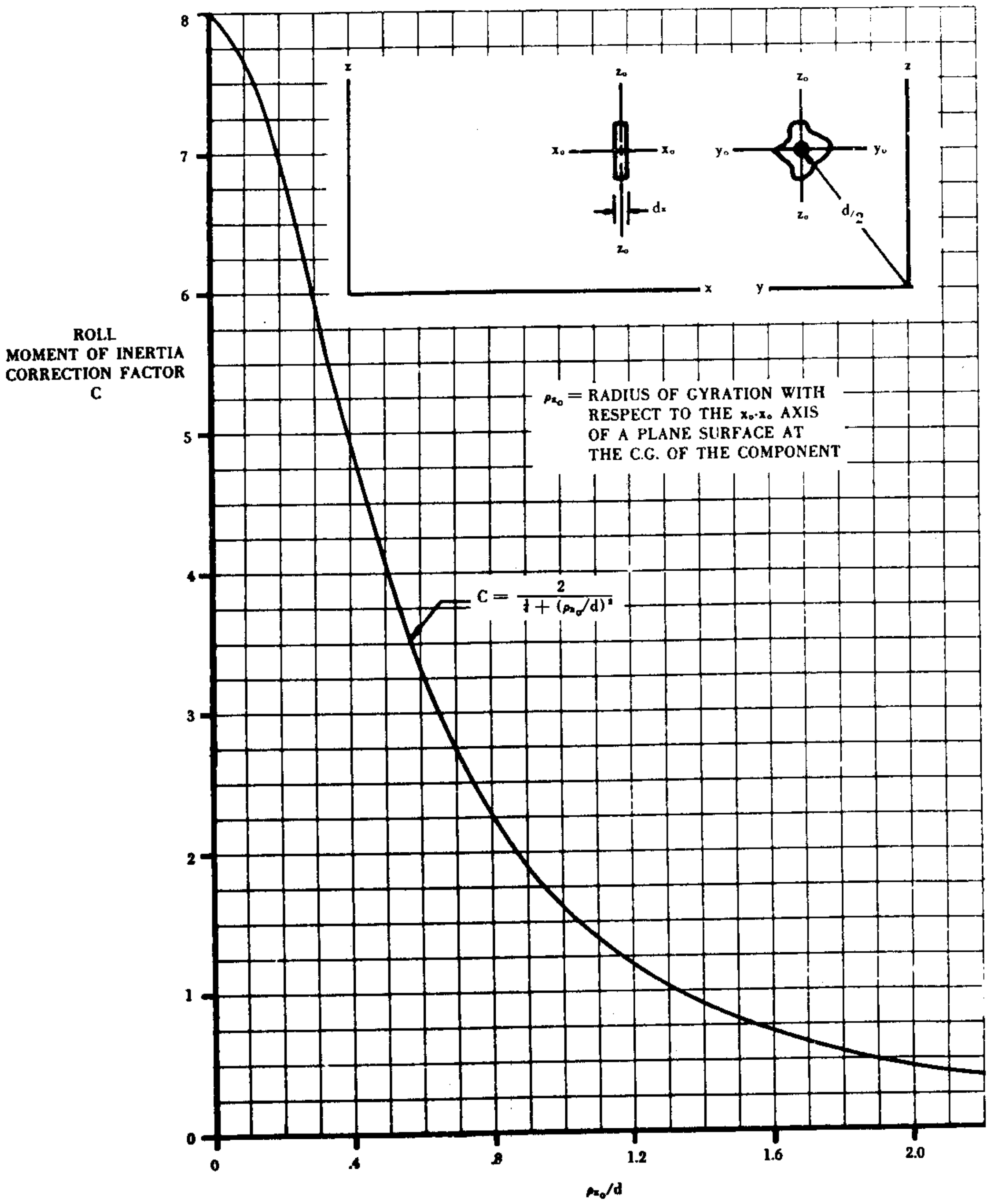


FIGURE 8.2-7 UNSYMMETRICAL SOLID COMPONENT
ROLL MOMENT OF INERTIA CORRECTION FACTOR C

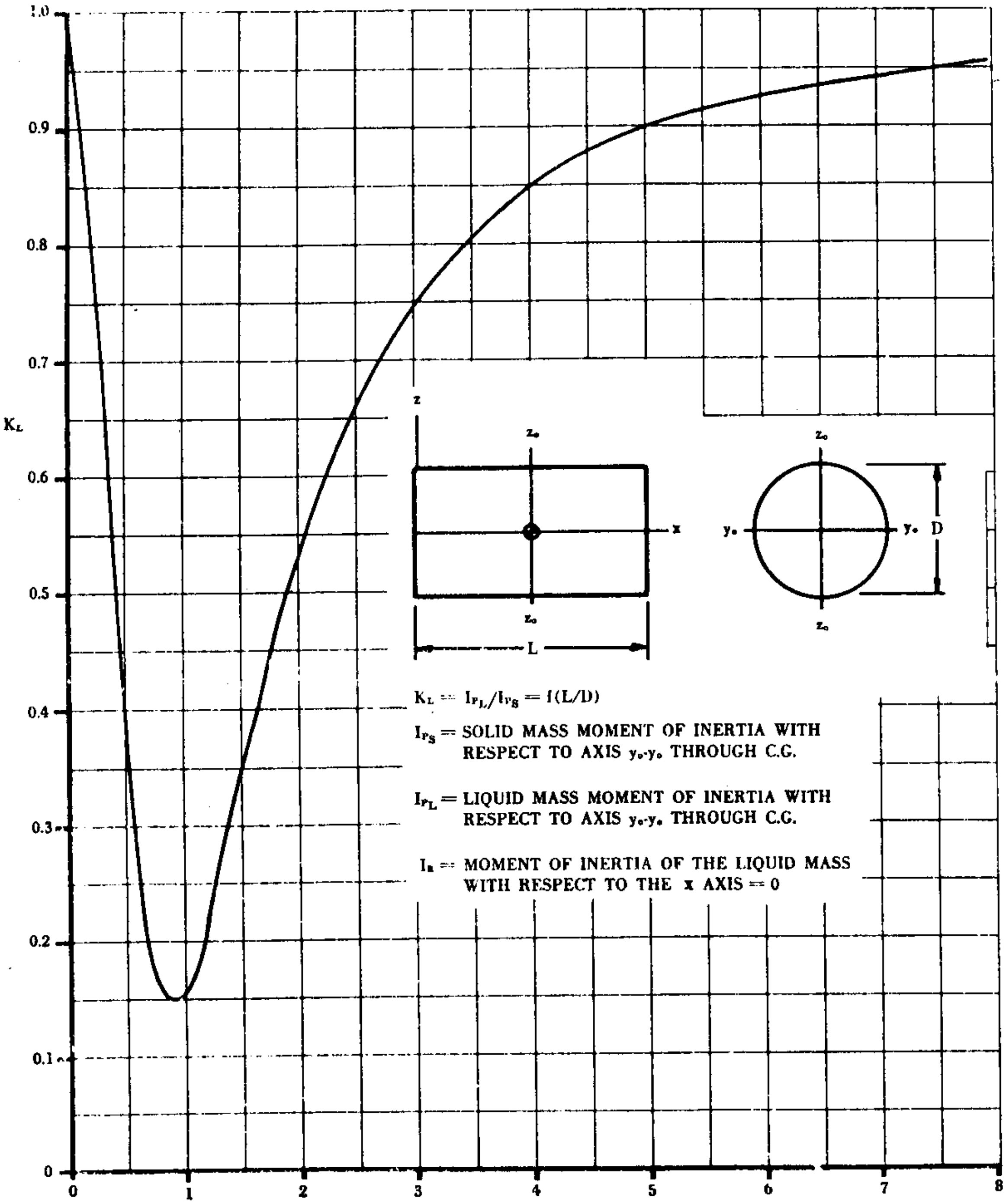


FIGURE 8.2-8 LIQUID MASS COMPONENT PITCH MOMENT OF INERTIA CORRECTION FACTOR K_L

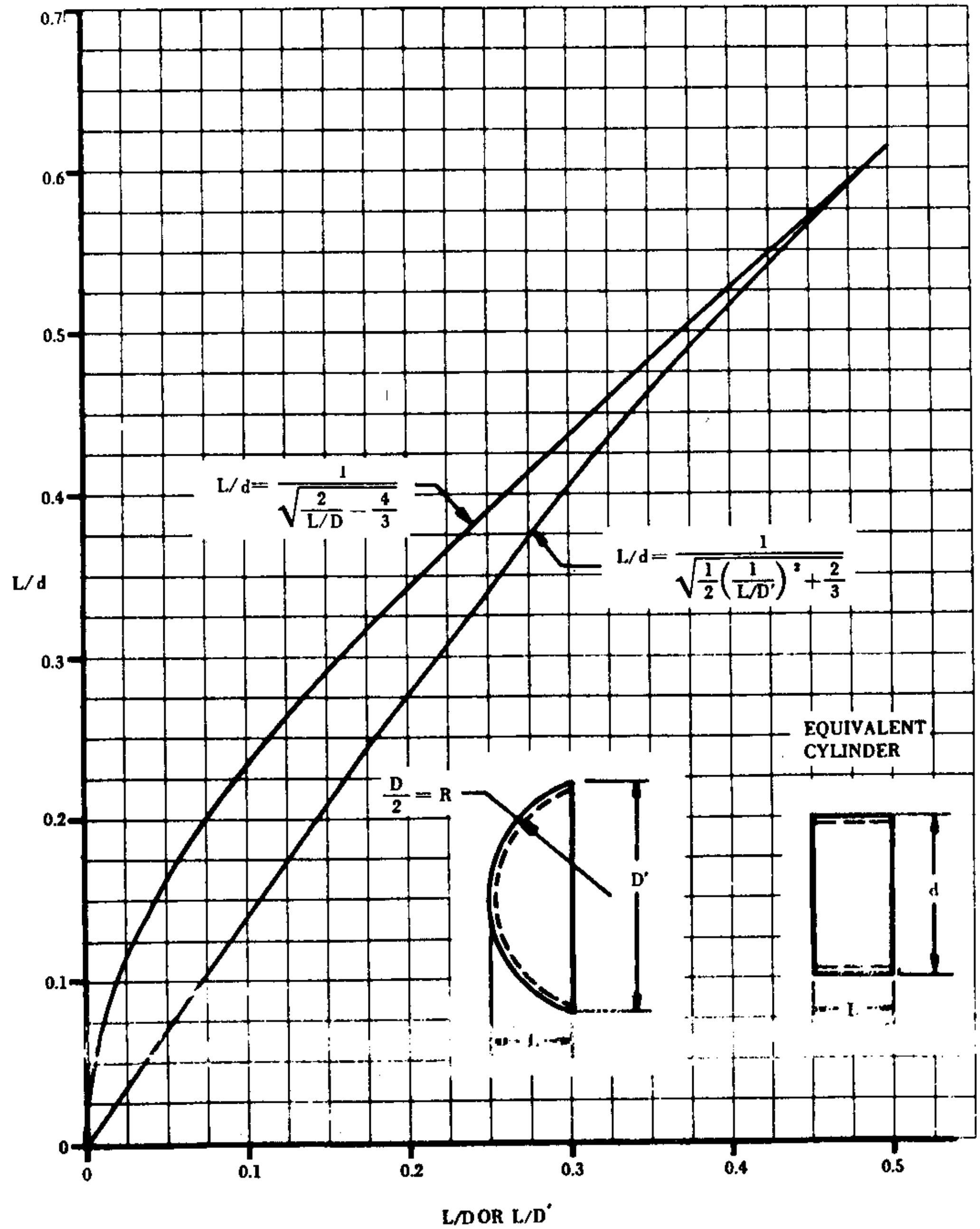
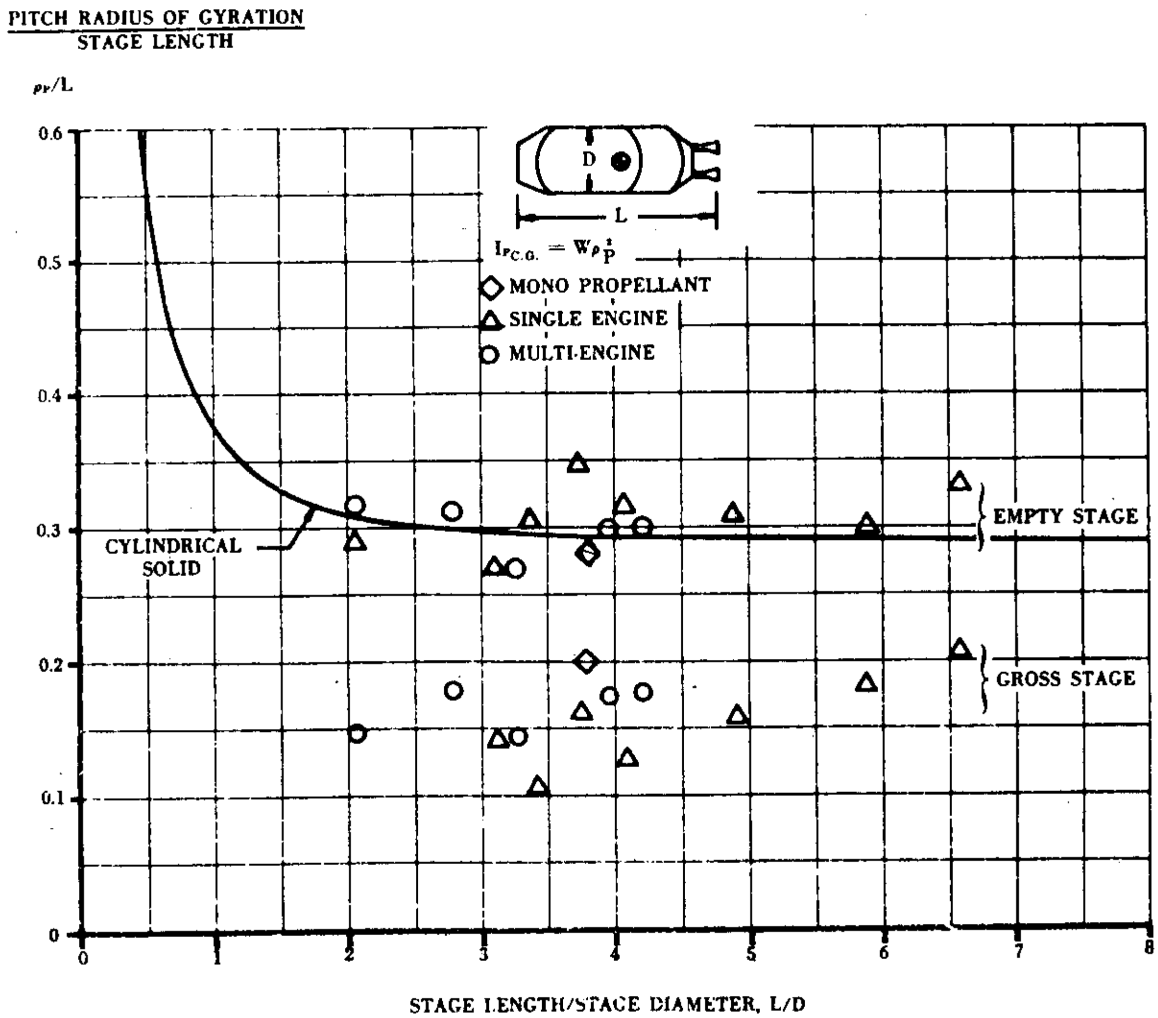


FIGURE 8.2-9 SPHERICAL SEGMENT SHELL EQUIVALENT CYLINDER FOR MOMENTS OF INERTIA



$I_{P_{C.G.}} = W \rho_P^2$

FIGURE 8.2-10 LIQUID PROPELLANT STAGE
PITCH RADIUS OF GYRATION

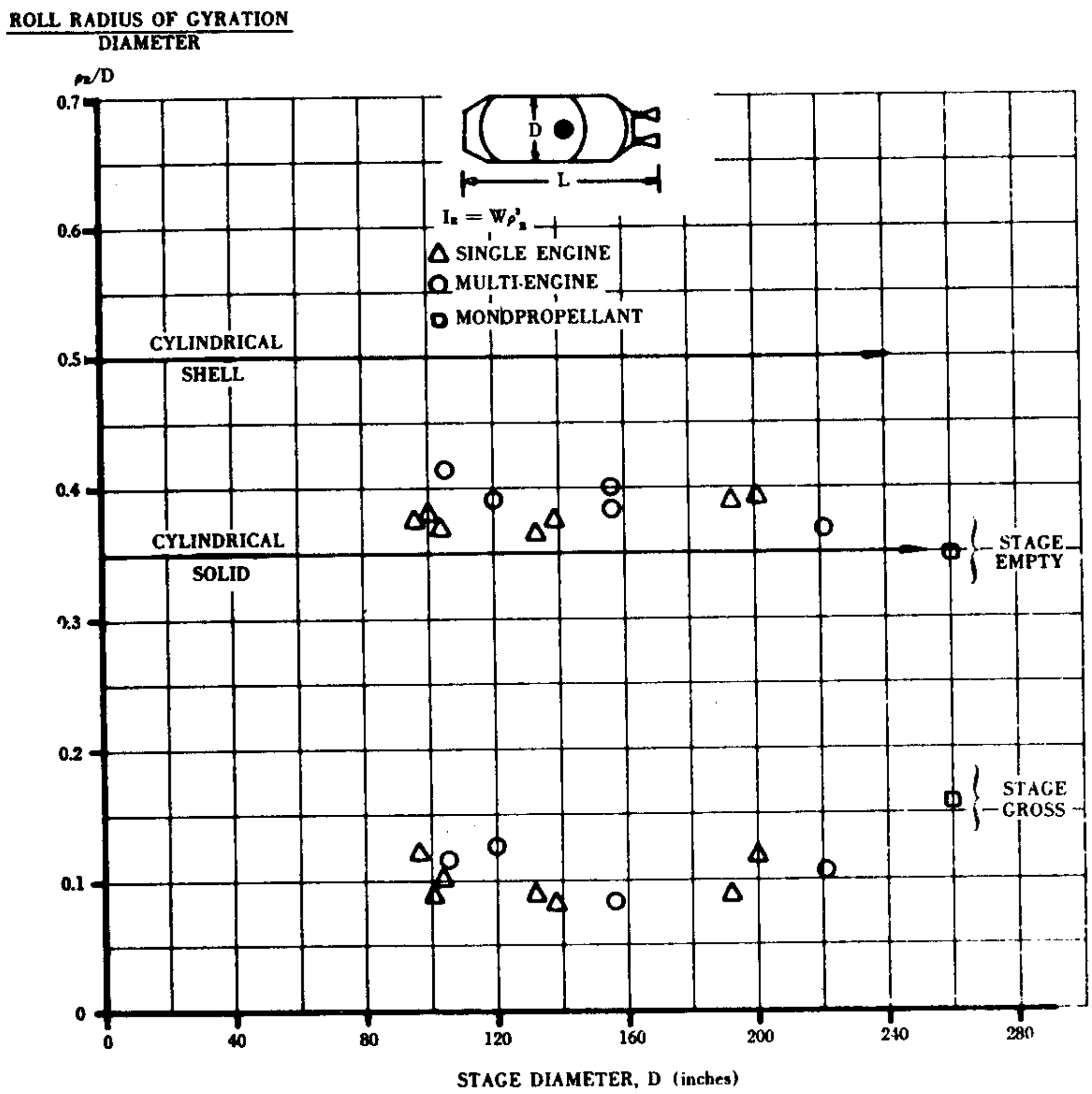


FIGURE 8.2-11 LIQUID PROPELLANT STAGE ROLL RADIUS OF GYRATION